Physics 321: HW 6

Problems 5.7, 5.10 in Sprott.

Hint for 5.10: The V(t) function shown is the product of two functions at the top left of the cheatsheet. Can you identify these and use the convolution theorem to find the Fourier transform of their product?

plus:

1) Suppose that we have a periodic waveform $V(t)$ with Fourier series coefficients of $\tilde{c}_n$. Find the Fourier series coefficients of $V(t - t_0)$ and $\frac{dV}{dt}$. $V(t\text{-}t_0)$ is the same function as $V(t)$ except that it has been shifted to the right by a time t_0.

Also prove that:

$$\int_T |V(t)|^2 dt = \sum_n |\tilde{c}_n|^2$$

2) Do the continuous transform version of 1) above. Suppose that we have a waveform $V(t)$ with Fourier transform $\tilde{V}(\omega)$. Find the Fourier transforms of $V(t - t_0)$ and $\frac{dV}{dt}$. Also prove that:

$$\int_{-\infty}^{+\infty} |V(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{V}(\omega)|^2 d\omega$$

(The physical interpretation of this in a circuit context is conservation of electrical energy, since power is proportional to V^2.)

Note: In terms of problem solving skills using Fourier transforms problems 5.7 and 5.10 are the most important here. 1) and 2) are exercises in the basic properties of Fourier transforms (the sort of thing you do when you cover FT's in a math course) and are intended to make the process less mysterious. You will have problems like 5.7 and 5.10 on exams in this course, but not the others.