

# Experiment 2

## DC Circuit Theorems

### 1 Motivation

The analysis of a circuit can often be simplified by finding a suitable *equivalent circuit* that will reproduce its behavior. For an ideal linear circuit, the equivalent behavior is exact for all currents and voltages. For a circuit that has nonlinear components, its behavior might be approximately linear for small variations near a specific *operating point*. The “small signal” behavior can then be represented as a linear equivalent circuit.

### 2 Background

A circuit is **linear** if, when you apply a voltage  $V$  at some port and measure the current  $I$  at another port with all other ports shorted,  $V/I$  is a constant as  $V$  is varied. (There are other equivalent definitions of a linear circuit). A single resistor  $R$  is a trivial linear component: if you apply a voltage  $V$ , a current  $V/R = I$  flows across it. The ratio  $V/I = R$  is therefore constant. In this lab we will investigate several useful theorems for linear circuits: Thevenin’s theorem, Norton’s theorem, the Superposition theorem, and the Reciprocity theorem.

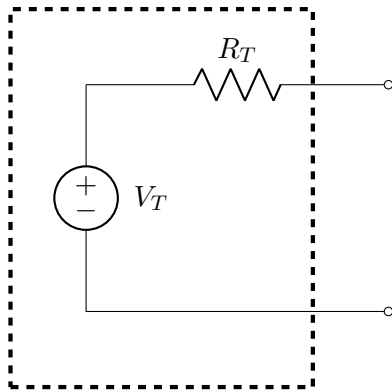
**Thevenin and Norton’s theorems** let us think of the inside of a “black box” circuit as a simple **equivalent circuit**, as shown in Figure 1. The component values of the imaginary equivalent circuit can be determined by measuring the voltage across the port (for Thevenin, called the “open-circuit voltage”  $V_{OC}$  and  $I_{SC}$ . If your measurement devices are ideal, the open-circuit voltage is  $V_T$  and the short-circuit current is  $I_N$ . The resistances  $R_T$  and  $R_N$  turn out to be equal, given by  $R_T = R_N = V_T/I_N$ . The **Superposition theorem** essentially says that inputs at each port produce outputs at the other ports independently of each other. The **Reciprocity theorem** says that the circuit is “invertible”: if we send an input to a port and measure an output at some other port, we can switch the input and output ports and get the same output from the same input.

Your tool in this lab for investigating these theorems is a “black box” circuit with three external ports. Each port has a pair of terminals that are connected to circuitry inside the box, analogous to the terminal pair in Figure 1. At the end of the lab you will open the box to see the circuit hidden inside. *The various DC circuit theorems will only hold if all unused ports of the box are shorted. Throughout the entire procedure below, make sure the unused ports are shorted!* The box also has a switch to select two modes for the circuit’s operation: **R** and **L**. By investigating the behavior of the box in these two modes, you will determine which mode is linear and which mode is nonlinear. At the end of the lab you will learn what caused the circuit to behave nonlinearly.

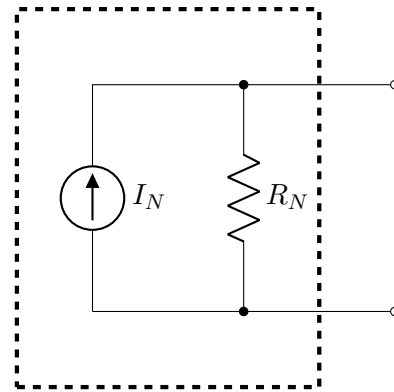
### 3 Equipment

For this lab, you will need:

- One preassembled “black box” circuit (record its number ID in your logbook)
- One Topward dual DC power supply, set for independent supplies (slide switches)
- One DMM4020 digital multimeter (voltmeter, ammeter)
- One ELC variable resistance box



(a) The Thevenin equivalent circuit



(b) The Norton equivalent circuit

Figure 1: The inside of the dashed box is really some complicated linear DC circuit, but from the outside of the box it appears to be either a single voltage source in series with a single resistor (Thevenin) or a single current source in parallel with a single resistor (Norton). Any measurements made at the terminals on the right side of each box will behave as if the inside of the box is either of the equivalent circuits.

## 4 Procedure

### 4.1 Thevenin's Theorem

With the black box's switch in the **R** position, short port #2 and apply a variable voltage  $V_1$  at port #1. Measure the current  $I_3$  that appears at port #3 for  $V_1$  ranging from 0 V to 20 V in 2 V steps. Plot  $I_3$  vs.  $V_1$ . Is the circuit **linear**? Does Thevenin's theorem apply? (By the way, why do we not explicitly instruct "short port #3"?)

Make the same measurements and answer the same questions with the box's switch in the **L** position.

### 4.2 Thevenin and Norton Parameters

Switch the box back to **R**. We will now determine  $V_T$ ,  $R_T$ , and  $I_N$  for port #3 being driven by port #1 using several different methods.

#### 4.2.1 Open-Circuit Voltage and Short-Circuit Current

Set  $V_1 = 20$  V. Measure the open-circuit voltage  $V_{OC}$  and the short-circuit current  $I_{SC}$  at port #3. Based on your measurements and Thevenin's and Norton's theorems, calculate  $V_T$ ,  $R_T$ , and  $I_N$ .

#### 4.2.2 Linearity

Attach a variable resistor  $R$  in series with an ammeter at port #3. Measure the voltage  $V$  across port #3 and the current through the resistor as  $R$  is varied. Varying  $R$ , take about ten measurements such that they are roughly evenly-spaced voltage steps up to maximum  $V$ . Tabulate your measurements for  $R$ ,  $V$ , and  $I$ . A maximum value of  $R$  around 10 k $\Omega$  should be about right. Make a plot of  $I$  vs.  $V$  and determine  $R_T$  from the slope of a line fit to the measurements. If you have a carefully drawn plot, drawing a line with a ruler will work.

### 4.2.3 Output Impedance

Remove the ammeter so that port #3 has just the resistor  $R$  connecting its terminals. Vary  $R$  until the voltage across the resistor (and thus port #3) is  $V_{OC}/2$ . This value of  $R$  equals  $R_T$ , because you have established a 50-50 voltage divider (by percentage) between the box and external  $R$ .

### 4.2.4 Direct Resistance Measurement

Disconnect the voltage source from port #1 and short its terminals. Disconnect the resistor  $R$ . Directly measure the resistance of port #3. By Thevenin's theorem this should equal  $R_T$ . Compare your four measurements of  $R_T$ , assuming the direct resistance measurement to be the most accurate.

## 4.3 Superposition Theorem

If the circuit is linear, it should obey the Superposition theorem. For example, if we apply voltages at ports #2 and #3, the current we observe at port #1 should be the sum of the currents we would observe with those voltages applied independently.

Simultaneously apply  $V_2 = 20\text{ V}$  and  $V_3 = 15\text{ V}$ , then measure the current at port #1. Then apply the input voltages independently and measure the independent currents (remember to short-circuit the disconnected ports). Does the Superposition theorem hold for this circuit?

Repeat the above with the switch in the **L** position. Does superposition hold now?

## 4.4 $G$ Parameters and Reciprocity

Because the circuit obeys Superposition, we can determine all of the currents if we are given all of the voltages, like so:

$$\begin{aligned} I_1 &= G_{11} V_1 + G_{12} V_2 + G_{13} V_3 \\ I_2 &= G_{21} V_1 + G_{22} V_2 + G_{23} V_3 \\ I_3 &= G_{31} V_1 + G_{32} V_2 + G_{33} V_3 \end{aligned} \tag{1}$$

Our goal is to measure all nine “ $G$  parameters” ( $G_{11}$ ,  $G_{12}$ , etc.).

To measure a specific  $G_{ij}$ , short the port that isn't  $i$  or  $j$  (so that port's voltage is zero), connect an ammeter to port  $i$  (so that  $V_i$  is also zero), and apply a voltage to port  $j$ . For example, suppose we want to measure  $G_{12}$ . The relevant equation is the first line of (1),

$$I_1 = G_{11} V_1 + G_{12} V_2 + G_{13} V_3 \tag{2}$$

We short port #3, so  $V_3$  is zero:

$$I_1 = G_{11} V_1 + G_{12} V_2 \tag{3}$$

Then we put an ammeter across port #1, so  $V_1$  is also zero and we can measure  $I_1$ :

$$I_1 = G_{12} V_2 \tag{4}$$

Now we just need to apply a known voltage  $V_2$ , measure  $I_1$ , and rearrange to calculate  $G_{12}$ .

Measure all nine  $G$  parameters of the box with the switch in the **R** position. If the circuit is linear, it should obey the Reciprocity theorem, which says that if put in a voltage at some port and measure a current at another port, we should be able to get the same current at the first port by putting the same voltage into the second port. If you work through the math, you will find that this implies that  $G_{ij} = G_{ji}$ . Are your measurements consistent with the Reciprocity theorem?

#### 4.5 Opening the Box

Now that you have extensively probed the box's behavior, it is time to look inside! Open the box and make a schematic diagram of the circuit you see when the switch is in the **R** position. You will need to use the color coding scheme to determine the labeled values of the resistors. Using the labeled values for components, evaluate  $V_T$ ,  $R_T$ , and  $I_N$  for the circuit you used in Section 4.2.1. Compare your measurements made previously to the values you just evaluated.

Look for the nonlinear component in the circuit. What is it, and why is it nonlinear?