PHY 321, 2021F

Experiment 3
Bridge Circuits

1 Motivation

This experiment explores using a DC Wheatstone Bridge to make precise resistance measurements. The lab equipment permits resistance measurements that have an accuracy of $\approx 0.5\%$. You will also use an AC Wheatstone Bridge to make an inductance measurement. You will employ error propagation methods to analyze the uncertainty in your measurements.

Before coming to the lab, study Appendix A (available on Canvas), which summarizes the nature of errors and error propagation. You can also prepare in advance answers for Step 1 in both the DC and AC bridge procedures.

2 Background

A Wheatstone Bridge circuit (Fig. 1) can be used to measure an unknown resistance (impedance more generally) in terms of a known resistance. This is because the bridge can be precisely "nulled" with one or more of the resistance values known precisely. It is often easier and more precise to measure zero than a finite value of voltage or current. The nulling method can eliminate or reduce extraneous interference, for example variations in the power supply voltage or 120 VAC pickup, which can induce measurement errors.

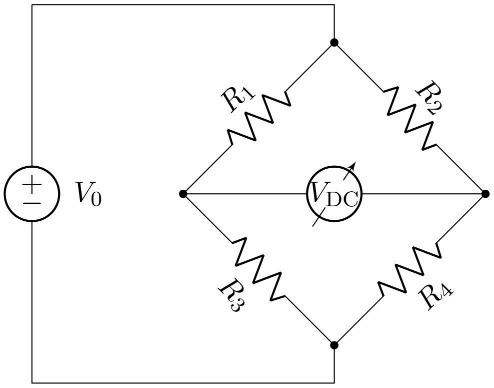

Figure 1: DC Wheatstone Bridge. Think of the bridge as made up of two voltage dividers. One divider is formed by R_1 and R_3, and the other is formed by R_2 and R_4. The voltmeter measures the potential difference between the midpoints of the two dividers. When this potential difference is zero, the bridge is said to be "nulled" or "balanced". It may be helpful to consider ground as the negative side of the voltage source. (The null condition can also be measured by a current meter, such as a mechanical galvanometer.)

3 Equipment

For this lab, you will use:

- One Topward dual DC power supply, set for independent supplies (slide switches)
- One DMM4020 digital multimeter (voltmeter, ohmmeter)
- Three ELC variable resistance boxes
- One "black box" that was used in *Experiment 2: DC Circuit Theorems*
- One AFG2021 Arbitrary Function Generator
- One ELC variable capacitor box
- One inductor coil mounted on a circuit board
- Optional: Commercial AC bridge for general impedance measurements

4 Procedure for the DC Wheatstone Bridge

1. Derive then summarize in your lab notebook the null (or balance) condition for the DC Wheatstone Bridge.
2. Construct the DC bridge circuit using two ELC variable resistor boxes. One box will be used for both R_1 and R_3, and the other box will be used for both R_2 and R_4. Note that the resistor decades are connected in series, and there are terminals available between the decades. Use the "×10 K" decade on its own to set $R_1 = R_2 = 10\,\mathrm{k\Omega}$. You will then use the "×1 K to ×1" decades as variable R_3 and R_4. To start, set the knobs so that $R_3 = R_4 = 9,955\,\Omega$. Use jumpers to complete the bridge as shown in Fig. 1 paying close attention to which terminals you need to use on the resistor boxes.

   Note: *ELC claims the overall accuracy of their decade resistance box is 1%, and the accuracy of each decade resistor is 0.5%.*

3. Set the DMM for DC voltage measurement and place its inputs across the terminals between the "×10 K" and "×1 K" decades of the resistor boxes. The DMM will measure the bridge voltage at the midpoints between the R_1-R_3 and R_2-R_4 voltage dividers. You can use post-it "correction tape" or regular post-its to label the resistors for reference.

   NEVER use ink, pencil, or markers of any sort to put labels on the surfaces of the lab equipment !! Remove your labels when you are finished with the lab.

4. Turn on and set the Topward DC voltage sources to 0 V. Connect either supply to the circuit as shown in Fig. 1. Slowly increase the voltage to establish $V_0 = 10\,\mathrm{V}$. It is possible to damage the resistors by driving too much current, causing them to overheat. *Do not exceed* ±10 V on the power supply. (They are capable of producing ±30 V.)
5. Perform an initial null of the bridge: Monitoring the DMM voltage measurement, adjust the knobs for R_3 and R_4 to make the voltage across the bridge as small as you can. Pay attention to the polarity! Make small adjustments first to get a feel for how the bridge responds. You may be compelled to make knob steps that land on 0 or 10, in which case you need to "cross" a decade to make further progress, or try adjusting the opposite resistor. You should be able to null the bridge to $< \pm 2\,\mathrm{mV}$ easily.
6. You can null the bridge further by making a fine-tune adjustment of R_3 (or R_4 by symmetry) that is more precise than the $\pm 1\,\Omega$ allowed by the resistor box. Do this by placing the 3rd ELC variable resistor box across (i.e., in parallel to) R_3 (or R_4). Note your new circuit in

your lab notebook. You could use the label R_5 for the resistor in parallel (or any descriptive label). Remember that the equivalent resistance of two resistors in parallel is less than either one. You might find it helpful to start with the 3rd decade box set to maximum resistance "(10)(10)(10)...". Making adjustments to the parallel combination, you should now be able to null the bridge $\lesssim 10\mu$V, and possibly zero to the limits of the DMM reading! When no further progress can be made, record the residual voltage across the bridge and the values of all resistors (according to their stated values on the boxes). Keep in mind the manufacturer's accuracy for the resistances.

7. Calculate the ratios R_1/R_3 and R_2/R_4 using the values of the resistor settings, taking into account the parallel combination for the fine-tuned R_3 (or R_4). Does $R_1/R_3 = R_2/R_4$ as expected? Calculate the uncertainty in each of the ratios using the manufacturer's specification by following the method described in Appendix A. Do your calculated ratios agree within expected uncertainty?
8. Estimate the *sensitivity* of the R_4/R_2 ratio using the sensitivity in the null. Start at null and make the smallest possible adjustment that causes the null residual to change sign. If you previously balanced the bridge as best possible, this will be one or two knob turns. Return to null and then adjust the other way to increase the magnitude of the residual voltage (same sign). Use the measured voltage difference, δV, of these step changes to estimate the sensitivity of R_4/R_2. You can use the variational analysis in Sprott section 2.6 to relate δV to δR_3, i.e., $\delta R_3/R_3 \approx 4\delta V/V$. How does the bridge's sensitivity compare with its accuracy as you determined above?
9. Now use the bridge to measure an unknown resistance. Install Port #3 of the "black box" in place of R_1 (or R_2 if you used R_4 for fine tuning in the previous steps.) You will be able to compare your measurement with your results from *"Exp 2: DC Circuit Theorems"* if you short the unused ports on the black box. Repeating the procedure above, null the bridge by making adjustments to R_3 (or R_4). Be sure not to change R_2 and R_4 (or R_1 and R_3), since they form a precise reference. Use the bridge null equation to predict the value of the black box resistance. Now measure the black box resistance directly using the DMM as an ohmmeter. Your measurement for the black box resistance may not be the same as the Thevenin equivalent resistance you found in *"Exp 2: DC Circuit Theorems"* because the boxes are not constructed using high precision components. (If you recorded the box number in your lab notebook and track down the same box, you could make a direct comparison with your Lab 2 results.)
10. Summarize your measurements in a table in your lab notebook, including estimated errors. The accuracy of the DMM can be found in its manual (see Files in Canvas).

5 Procedure for the AC Wheatstone Bridge

1. Derive then summarize in your lab notebook that the AC bridge in Fig. 2 is nulled (balanced) if $R_4 = R_2R_3/R_1$ and $L = R_2R_3C$. The reduction methods used for DC circuits still work, but reactive impedances are complex, $Z_L = j\omega L$ and $Z_C = 1/j\omega C$, while $Z_R = R$. You must manipulate them as complex numbers. Since the balance condition does not depend on ω, any frequency will work in principle. However, the measurement will be imprecise if the voltage across L is small. Here you will use an AC frequency that makes the magnitudes of the impedances comparable, $|Z_L| \sim |Z_{R_4}|$.
2. Construct the bridge with $R_1 = R_2 = 100\,\Omega$ using the "×100" decade and $R_3 = R_4 = 105\,\Omega$ using the "×10" and "×1" decades. The inductor mounted on a circuit board serves as an unknown impedance. Install it and the ELC adjustable capacitance box as shown in Fig. 2.

Set C to midrange on all decades "55555". Set the DMM to read AC voltage. By default, the DMM displays the rms AC voltage. Use the function generator to drive the bridge with a sine wave with frequency, $f = 1\,\mathrm{kHz}$. Set the output voltage of the generator to $5\,\mathrm{V}$ peak-to-peak. The DC offset voltage should be set to zero. Enable the generator's output using the yellow "On/Off" button on the front panel.

Note: *ELC claims the accuracy of their decade variable capacitance box is 1%.*

3. Null the bridge by adjusting C. Because of noise pickup, nulling to $\lesssim$ 10-30 mV rms may be the best you can do. Why does the measurement not cross zero like it did for the DC bridge? Change f by a factor of 2 (up and down) to verify that the null condition does not depend on frequency. When nulled, the bridge voltage should not change much when changing the frequency. When no further progress can be made, record your measurements and component values and estimate L using the null condition equation.
4. Estimate the sensitivity of the AC bridge by varying C and observing the change in null condition much like you did in Step 8 for the DC bridge. With $f = 1\,\mathrm{kHz}$, vary C from its value at null and note how much change, δC, is required to unbalance the bridge by a barely discernible amount. How does the bridge's sensitivity in measuring L compare with the uncertainty associated with the stated accuracies of C, R_2 and R_3?
5. Optional: Consult your lab instructor to see if a commercial, high-precision AC bridge instrument is available in the lab for an independent measurement of L. A commercial bridge uses the same principles, optimized over a range of parameters and can give simultaneous measurements of the resistance and reactance for any circuit component.

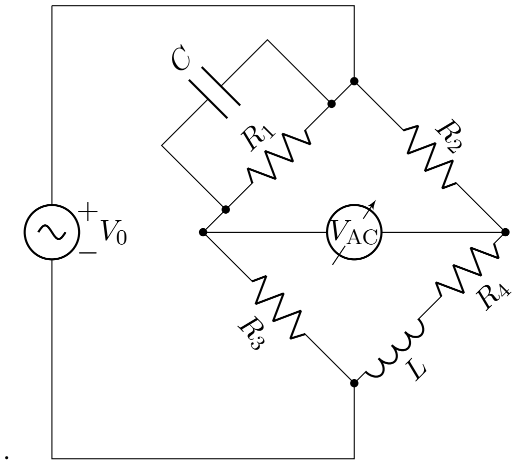

Figure 2: AC Wheatstone Bridge. The bridge has two voltage dividers with reactive elements driven by an AC sinusoidal voltage source at constant frequency, ω. The AC voltmeter measures the potential difference between the midpoints of the two dividers. As for the DC case, the bridge is "nulled" or "balanced" when this potential difference is zero. (What do you think happens if the voltage source has a DC component?)