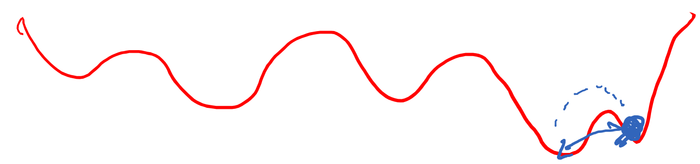
TELEGRAPH NOISE



TWO-STATE SYSTEM

MEAN DWELL TIME τ

IN GENERAL, $\tau_u \neq \tau_d$

RECALL, $S(f) = \int \langle v(t)\, v(0) \rangle\, e^{-2\pi j f t}\, dt$

$$\langle v(t)\, v(0) \rangle = +1 \cdot P(\text{EVEN}) - 1\, P(\text{ODD})$$

$$P_n(\lambda, t) = e^{-\lambda} \frac{\lambda^n}{n!}$$ POISSON DISTRIBUTION

λ = MEAN # OF TRANSITIONS

$$\langle v(t)\, v(0) \rangle = e^{-\lambda} \left\{ \sum_{n\ \text{EVEN}} \frac{\lambda^n}{n!} - \sum_{n\ \text{ODD}} \frac{\lambda^n}{n!} \right\} = e^{-2\lambda}$$

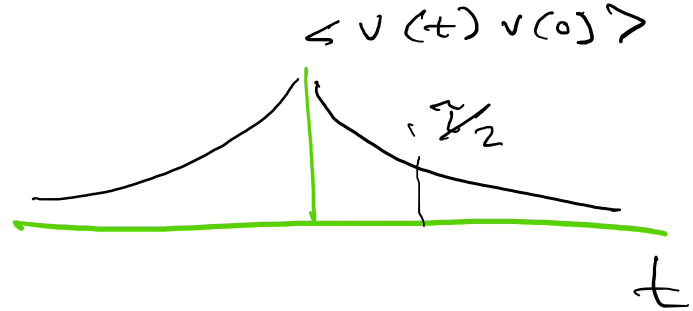
$$\sum \frac{(-1)^n \lambda^n}{n!}$$

$$\lambda = t/\tau$$

$\lambda = t/\tau$

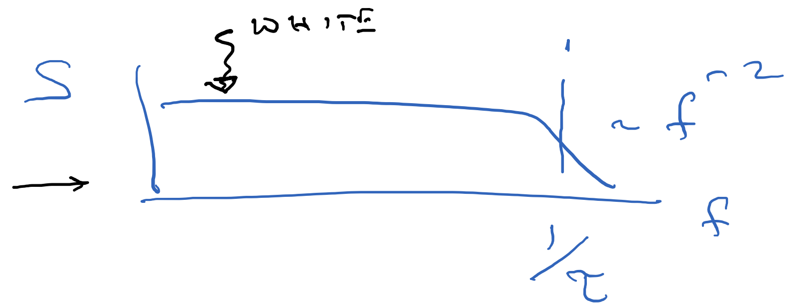
MEAN # OF TRANSITIONS IN TIME t

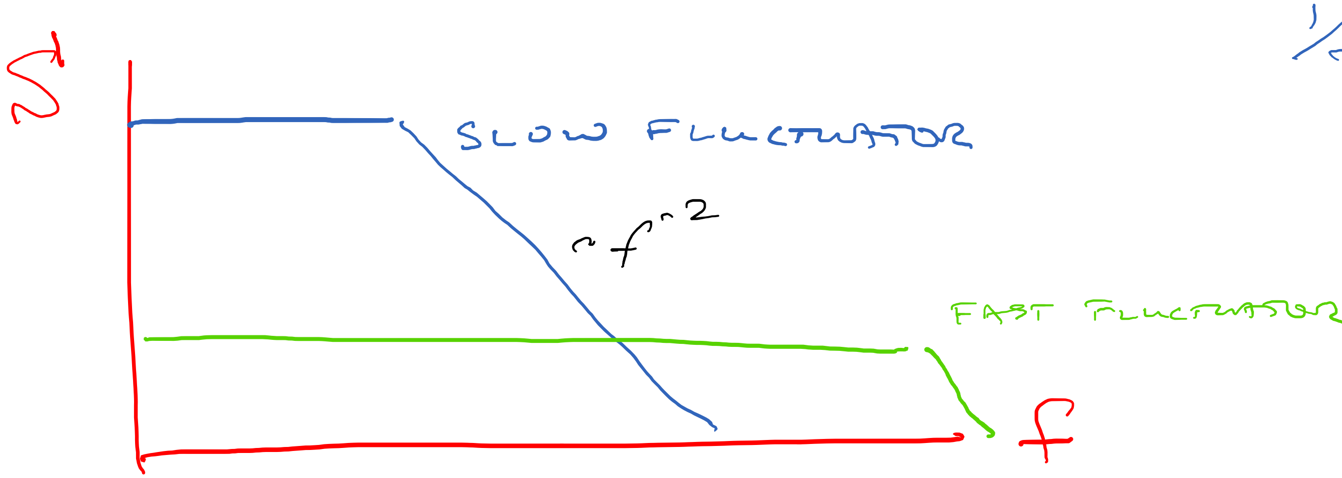
$$\therefore \langle v(t)\, v(0) \rangle = e^{-2t/\tau}$$



$$\rightarrow S(f) = \int e^{-2t/\tau} e^{-2\pi j f t}\, dt$$

$$\rightarrow S(f) = \frac{\tau}{1 + 4(2\pi f \tau)^2}$$

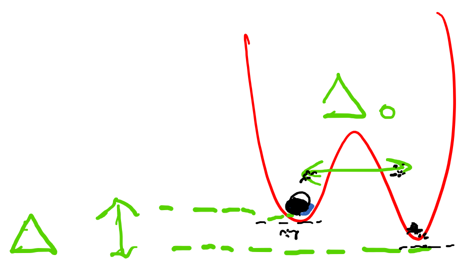




1/f NOISE "PINK NOISE"

DISTRIBUTION OF TWO LEVEL STATE (TLS) DEFECTS

"DUTTA-HORN" ?

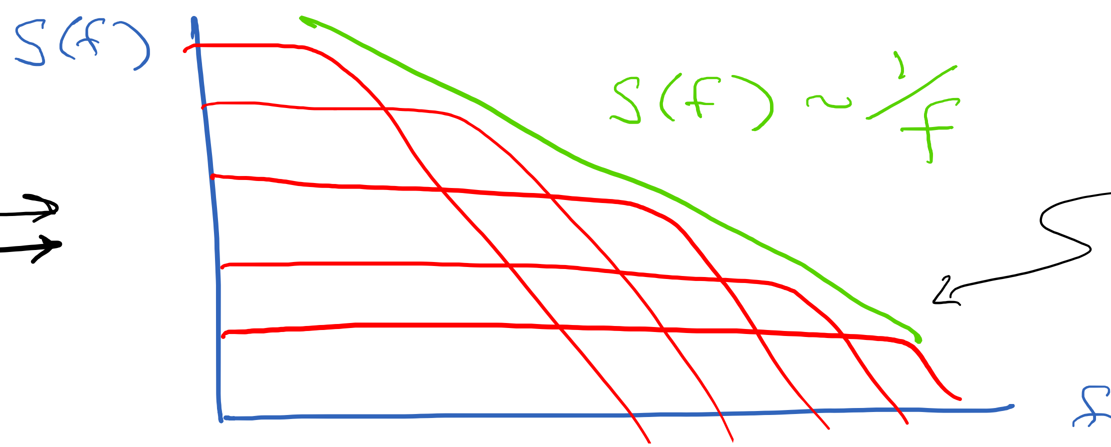


DISTRIBUTION

$$P(\Delta;\Delta_0) \sim \frac{1}{\Delta_0}$$

$$\int_{\Delta_{MIN}}^{\Delta_{MAX}} \frac{1}{\Delta_0}\, d\Delta_0 \sim \ln \frac{\Delta_{MAX}}{\Delta_{MIN}}$$

$\Rightarrow$ [UNIFORM DISTRIBUTION IN $\log f$]

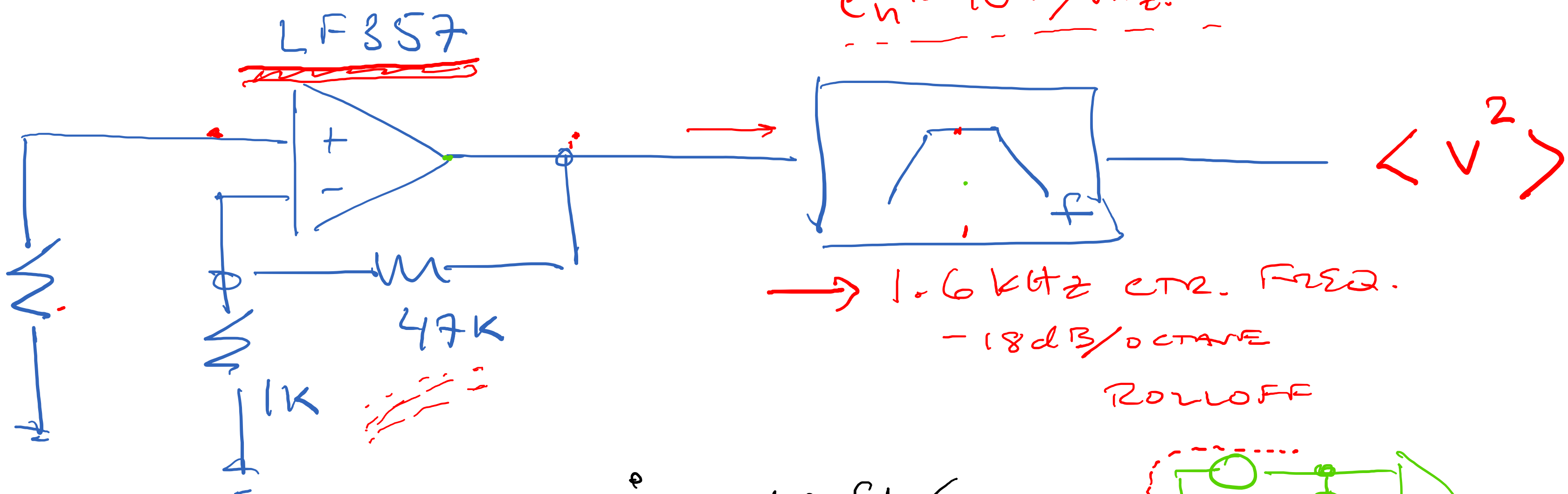


NOISE INCREASES @ LOW FREQUENCY

$\longrightarrow$ STAY AWAY FROM $f = 0$ FOR MEASUREMENT?

NOISE LAB

$S_V = 4k_BTR$



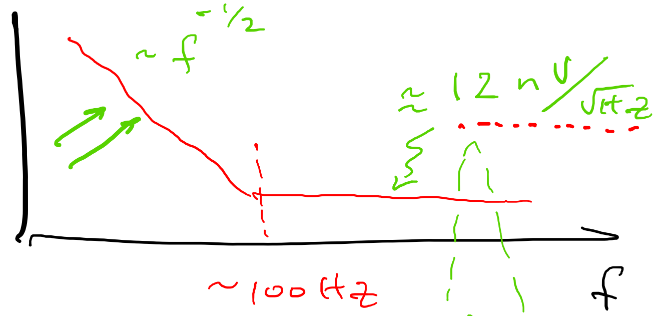
→ LM741

$e_n \sim 40\ nV/\sqrt{Hz}$

→ 1.6 kHz CTR. FREQ.

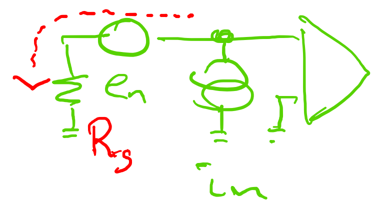
−18 dB/OCTAVE

ROLLOFF



$e_n \equiv S_V^{1/2}$

$i_n = 10\ fA/\sqrt{Hz}$

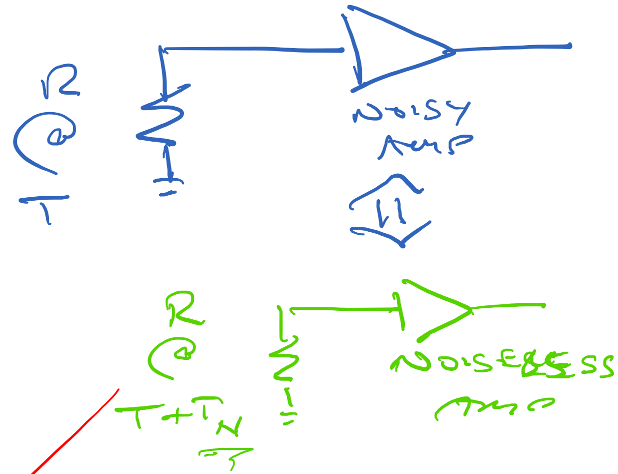


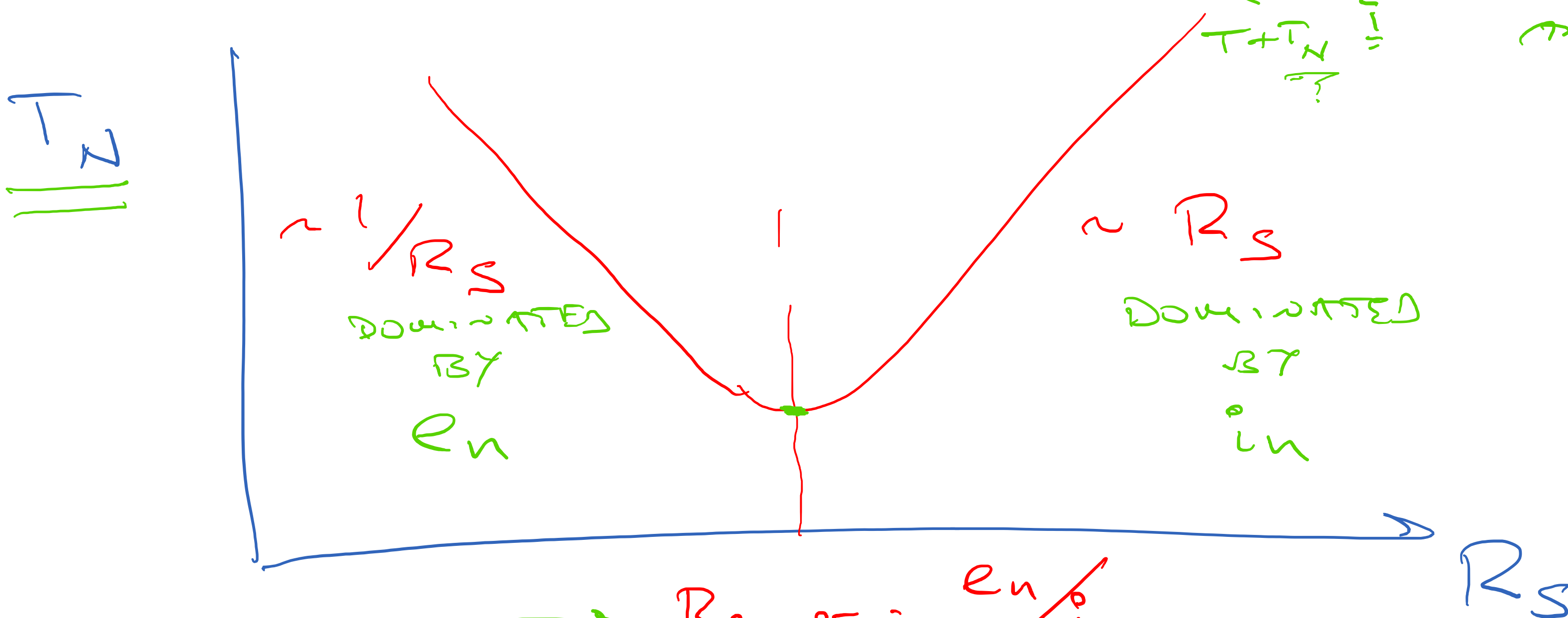
$$R_n = \frac{e_n}{i_n}$$

$$= \frac{10^{-8}\ V/\sqrt{Hz}}{10^{-14}\ A/\sqrt{Hz}} \sim 10^{6}\ \Omega.$$

NOISE TEMPERATURE

$$T_N = \frac{e_n^2 + R_S^2\, i_n^2}{4 k_B R_S}$$





$$R_{n,OPT} = e_n / i_n$$

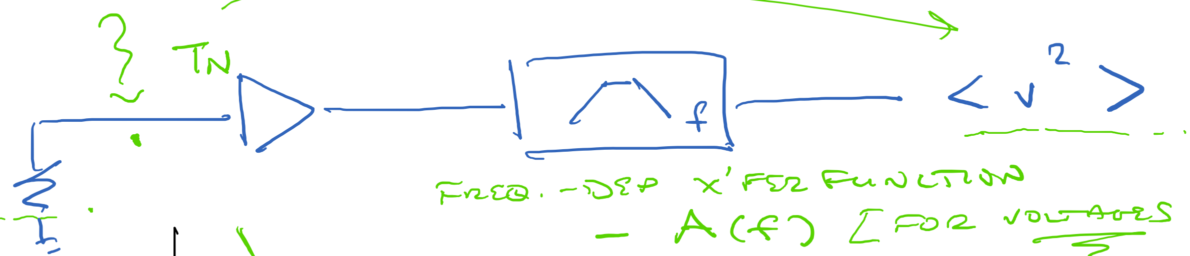
NOISE BANDWIDTH.

$$\langle v^2 \rangle = \int S_v(f)\,df$$

1 MΩ @ 300K

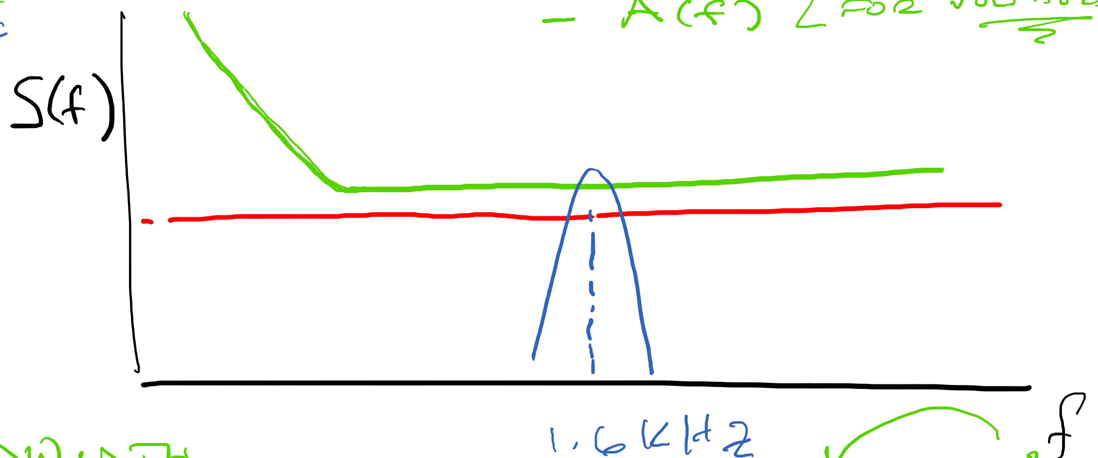
↓

~130 nV/$\sqrt{\text{Hz}}$



FREQ.-DEP X'FER FUNCTION

— A(f) [FOR VOLTAGES, NOT POWER].



NOISE BANDWIDTH

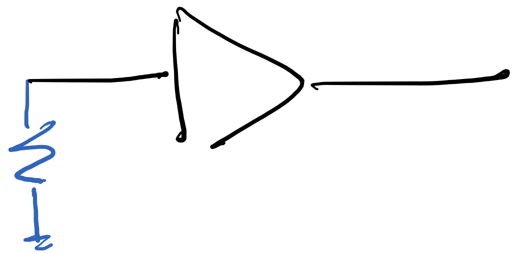
$$\Delta f_N = \int |A(f)|^2\,df$$

$$\langle v^2 \rangle = \int \left[4k_B(T+T_N)R\right] |A(f)|^2\,df$$

$$= 4k_B(T+T_N)R\,\Delta f_N$$

HOT LOAD / COLD LOAD

R @ T_C

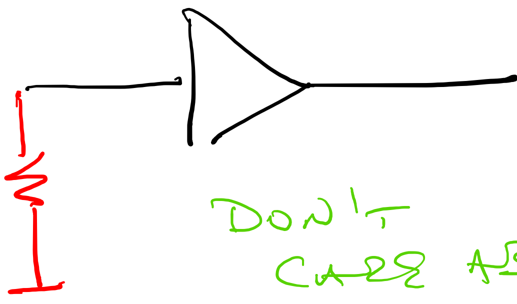


P_C

$\langle V^2 \rangle$ or S_ν

$$\frac{T_C + T_N}{T_H + T_N} = \frac{P_C}{P_H}$$

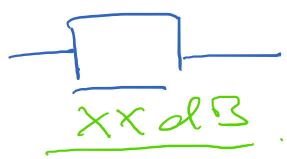
R @ T_H

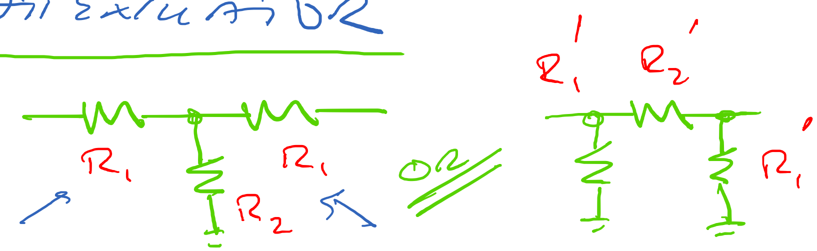


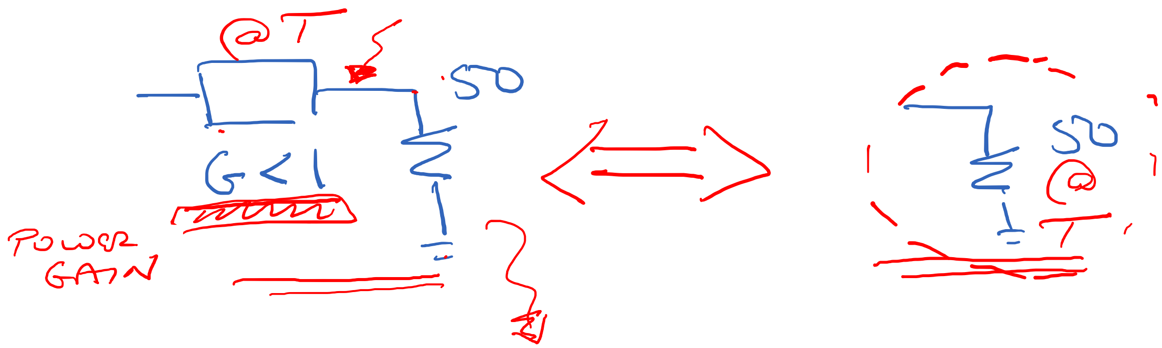
P_H

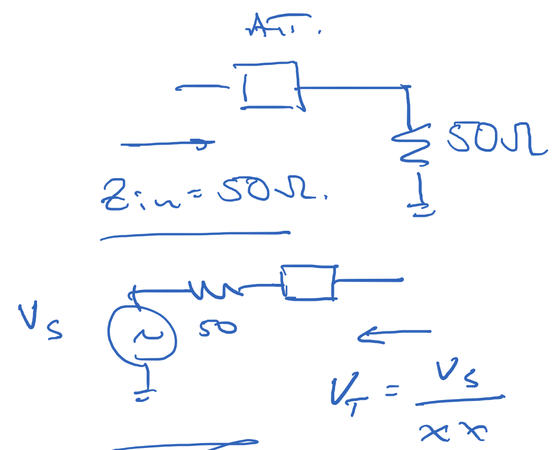
DON'T CARE ABOUT $\rightarrow A(f)$!

NOISE TEMPERATURE OF ATTENUATOR





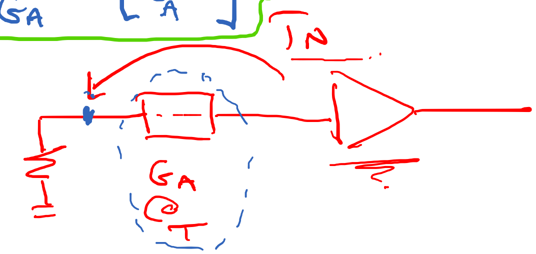


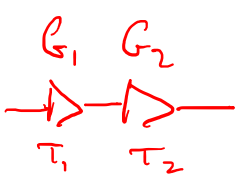


$$T = (T + T_N)\,G$$

$$\Rightarrow \quad T_N = \left[\frac{1}{G} - 1\right] T$$

$$\boxed{\frac{T_N}{G_A} + \left[\frac{1}{G_A} - 1\right] T}$$

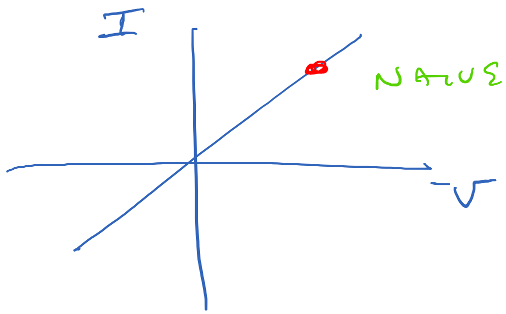




$$T_{N,\,TOTAL} = T_1 + \frac{T_2}{G_1}$$

PHASE DETECTOR

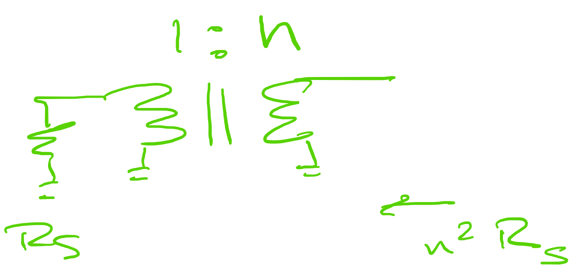
e.g. RESISTANCE THERMOMETRY w/ LOW-LEVEL EXCITATION



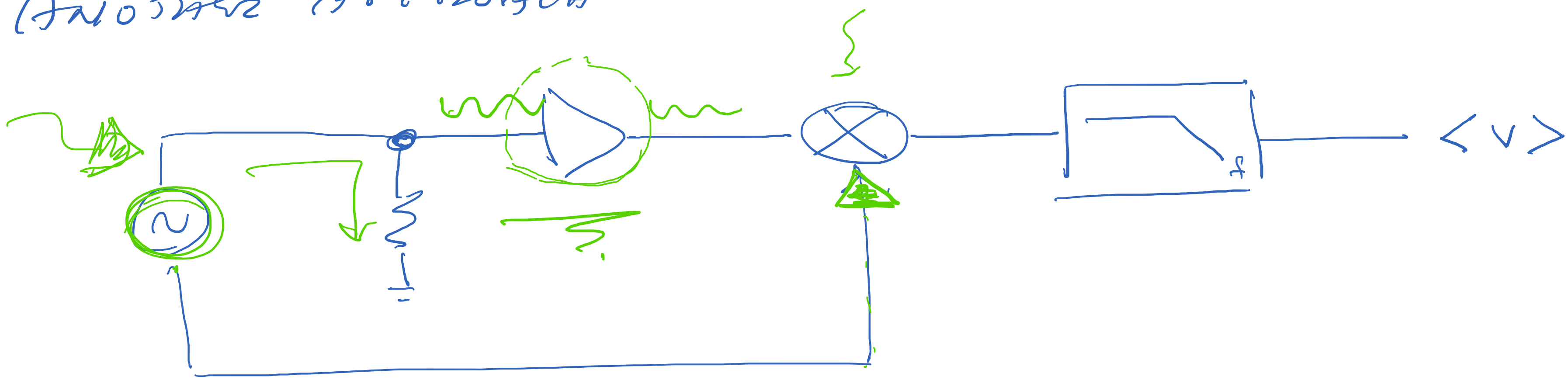
NAIVE APPROACH: BIAS w/ CURRENT, MEASURE VOLTAGE

PROBLEMS:

1. 1/f NOISE OF AMP
2. LOW-FREQUENCY DRIFT
3. $R_S \neq R_{N,\,OPT.}$

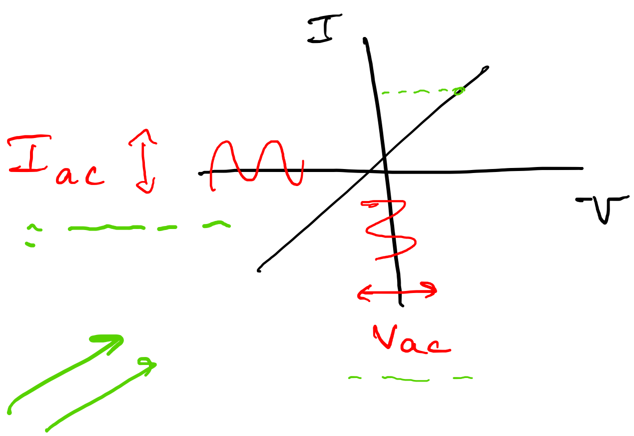


ANOTHER APPROACH



MODULATE EXCITATION @ NONZERO FREQUENCY f_0

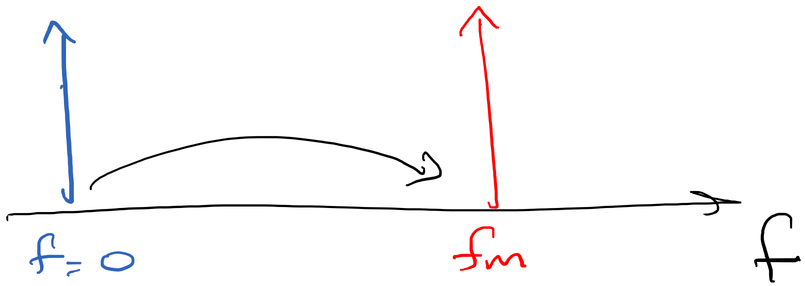
→ DETECT IN NARROW BAND AROUND f_0



→ $V_{AC} = R_{DYN} \cdot I_{AC}$

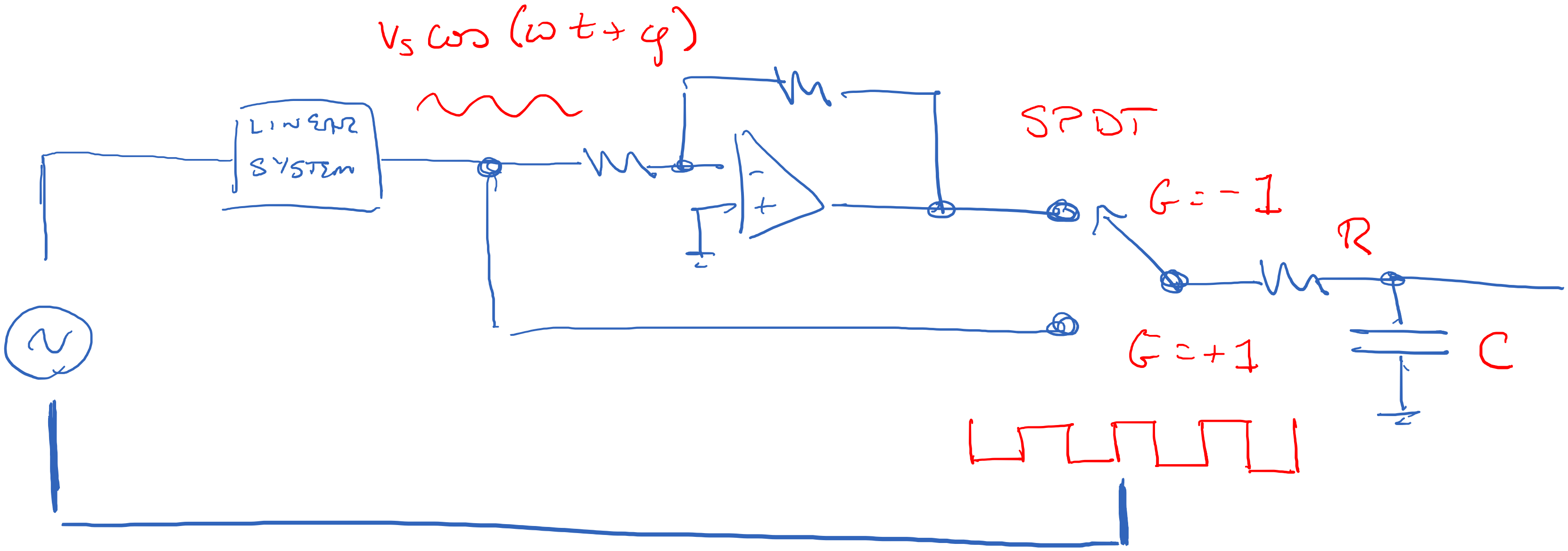
$V(t) = R_{DYN} \cdot I_0 \cos \omega_m t$

WHAT DOES THIS ACCOMPLISH?

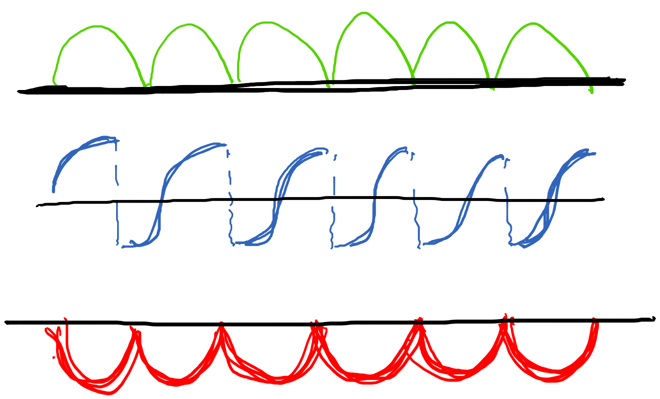


→ ESCAPE 1/f NOISE

ALSO, CAN NOW TRANSFORM SOURCE IMPEDANCE TO OPTIMALLY NOISE MATCH.



POSSIBILITIES:



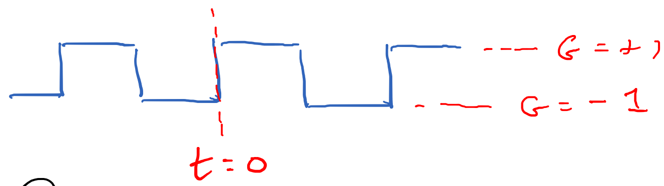
IN PHASE $\langle V_o \rangle > 0$

90° SHIFT $\langle V_o \rangle = 0$

180° SHIFT $\langle V_o \rangle < 0$

Assume $RC \gg 2\pi/\omega_m$ $\longrightarrow$ Just look @ DC part of output

REF.



Look @ $\langle v_s \cos(\omega t + \varphi) \rangle \Big|_0^{\pi/\omega} - \langle v_s \cos(\omega t + \varphi) \rangle \Big|_{\pi/\omega}^{2\pi/\omega}$

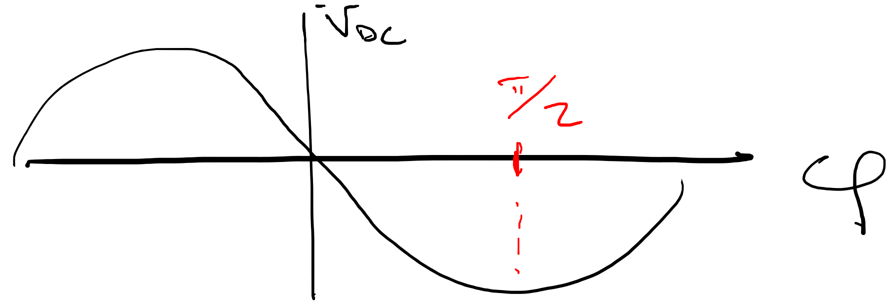
$$= \frac{\omega}{2\pi} v_s \left\{ \int_0^{\pi/\omega} \cos(\omega t + \varphi)\, dt - \int_{\pi/\omega}^{2\pi/\omega} \cos(\omega t + \varphi)\, dt \right\}$$

$$= \frac{v_s}{2\pi} \left\{ \sin(\omega t + \varphi) \Big|_0^{\pi/\omega} - \sin(\omega t + \varphi) \Big|_{\pi/\omega}^{2\pi/\omega} \right\}$$

$$= \frac{v_s}{2\pi} \left\{ 2 \sin(\pi + \varphi) - \sin\varphi - \sin(\varphi + 2\pi) \right\}$$

$$\boxed{V_{DC} = -\frac{2 v_s}{\pi} \sin\varphi}$$

$$V_{DC} = -\frac{2v_s}{\pi}\sin\varphi$$



FOR $\omega_s \neq \omega_m$

$$v_{out} = -\frac{2v_s}{\pi}\sin\Delta\omega t$$