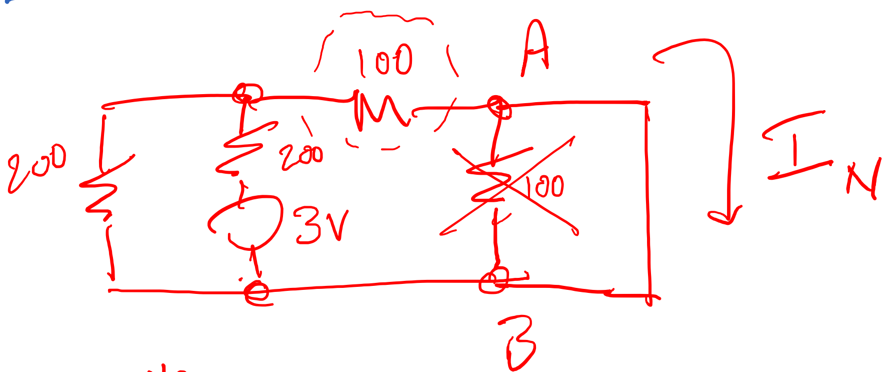
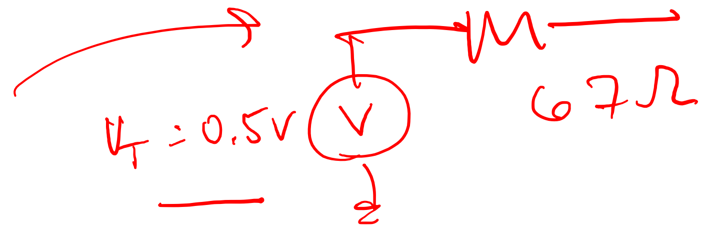
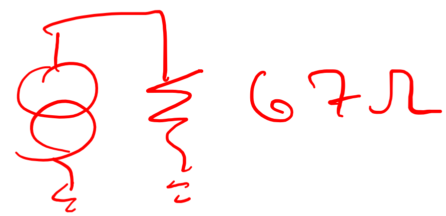
Questions ?

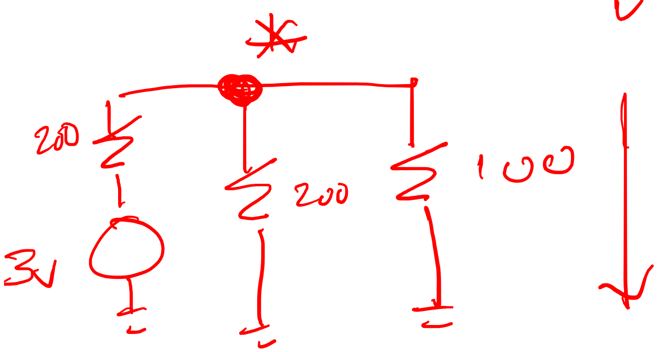






$$I_N = \frac{V_T}{R_T}$$

$$\frac{\frac{3}{4}}{\frac{4}{3} \cdot 100} = \frac{\frac{1}{2}}{} \quad 7.5mA\ ?$$



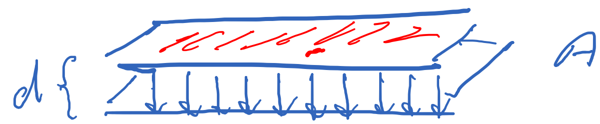
$$V_* = 3V \cdot \frac{\frac{2}{3}}{2 + \frac{2}{3}} = \frac{3}{4} V$$

$$\boxed{I_N = 7.5mA}$$

IMPEDANCE

CAPACITOR: CIRCUIT ELEMENT THAT STORES CHARGE / ELECTRIC FIELD ENERGY

PROTOTYPICAL:



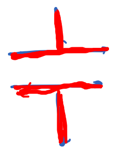
MAXWELL $\nabla \cdot \bar{E} = \frac{1}{\epsilon}\rho \qquad \int \bar{E} \cdot d\bar{A} = \frac{1}{\epsilon}\int \rho \, dV$

$$E \cdot A = \frac{1}{\epsilon} Q \Longrightarrow \frac{V}{d} \cdot A = \frac{1}{\epsilon} Q$$

$$\underline{Q = \frac{\epsilon A}{d} V} \Longrightarrow Q = CV, \text{ with } \underline{C = \frac{\epsilon A}{d}}.$$

UNITS: FARAD

DIFFERENTIATE: $$\underline{I = C\frac{dV}{dt}}$$ [DISPLACEMENT CURRENT]

SYMBOL 

OPEN @ DC
SHORT @ HIGH. FREQ.

$$\underline{\epsilon_0 \sim 8 \cdot 10^{-12}\ F/m}$$

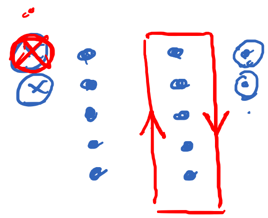
PRACTICAL CAPS: ~fF [MICROFAB]
~pF FOR RF
~mF FOR FILTERING/POWER APPLICATIONS

STORED ENERGY

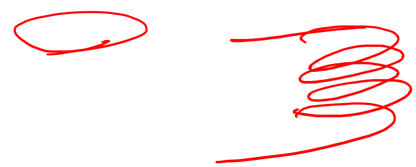
$$\int (IV)\, dt = C \int V \frac{dV}{dt}\, dt = \underline{\frac{1}{2} C V^2} = \underline{\frac{Q^2}{2C}}$$

INDUCTOR: ELEMENT FOR STORING MAGNETIC ENERGY [FLUX, FIELD]

PROTOTYPICAL:



LENGTH ℓ, n TURNS



MAXWELL

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

$$\int_A \longrightarrow \oint \bar{B} \cdot d\bar{\ell} = \mu_0 I \cdot n$$

$$B \cdot \ell = \mu_0 I n$$

$$B = \mu_0 \frac{n}{\ell} I$$

$$\mu_0 = 4\pi \times 10^{-7} \; H/m$$

$$\sim 10^{-6} \; H/m$$

$$\Phi = nBA = \mu_0 n^2 \frac{A}{\ell} I$$

DEFINE INDUCTANCE $L = \dfrac{\Phi}{I} \Rightarrow L = \mu_0 n^2 \dfrac{A}{\ell}$

INDUCTANCE GOES AS n^2

- PROPORTIONAL TO LENGTH
- UNIT: HENRY

$\mu_0 \approx 10^{-6}\ H/m$

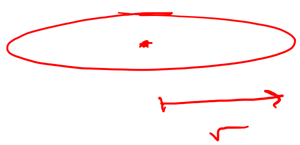
PRACTICAL INDUCTORS:

pH [MICROFAB. DEVICES]
nH [RF APPLICATIONS]
mH [FILTERS]

1 pH/μm

1 nH/mm

USEFUL:



$$L \simeq \mu_0 r$$

STORED ENERGY

SINCE $\Phi = LI$, $\quad V = L\frac{dI}{dt}$

$$\int (IV)\,dt = L\int I\,dI = \frac{1}{2}LI^2 = \frac{\Phi^2}{2L}$$

SYMBOL 

SHORT @ DC

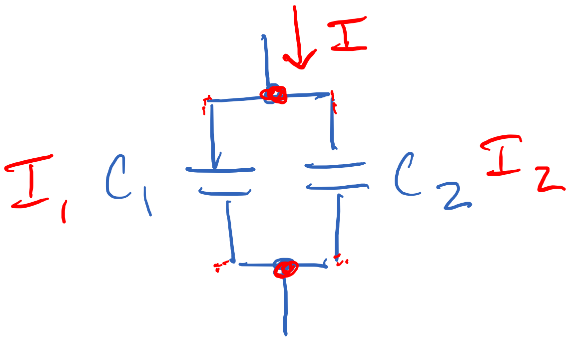
OPEN CIRCUIT @ HIGH FREQ.

DUALITY

$$\left\{\begin{matrix} Q \longleftrightarrow \Phi \\ C \longleftrightarrow L \\ V \longleftrightarrow I \\ E \longleftrightarrow B \end{matrix}\right\}$$

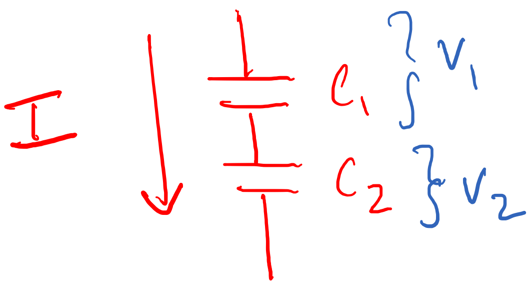
QUANTUM MECHANICALLY, $\hat{\Phi}$ AND $\hat{Q}$ ARE CONJUGATE VARIABLES

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$



$$I_1 = C_1 \frac{dV}{dt} \; ; \; I_2 = C_2 \frac{dV}{dt} \; \ldots$$

$$I = (C_1 + C_2)\frac{dV}{dt} \Rightarrow \underline{\underline{C_{eff} = C_1 + C_2}}$$



$$I = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt} \; \ldots$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

$$\dot{V} = \frac{I}{C_1} + \frac{I}{C_2} = \frac{1}{C_{eff}} \cdot I$$

$$\longrightarrow \; C_{eff} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1}$$

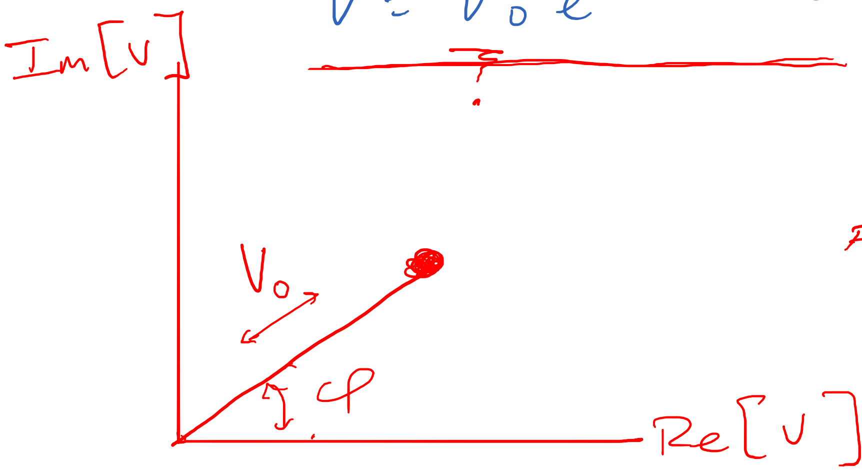
FREQ.-DEPENDENT CIRCUITS, IMPEDANCE

CONSIDER $V = V_0 \cos(\omega t + \varphi)$

WRITE AS $V = V_0 e^{j(\omega t + \varphi)}$

PHASOR

$j \equiv \sqrt{-1}$

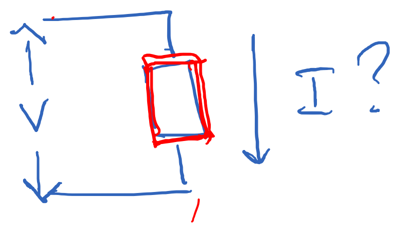


FRAME ROTATES @ ANG. FREQ. ω

@ END OF DAY, TAKE REAL PART TO DETERMINE PHYSICAL VOLTAGES, CURRENTS

$$\mathrm{Re}\left[V_0 e^{j(\omega t + \varphi)}\right] = V_0 \cos(\omega t + \varphi)$$

TAKE $\underline{V = V_0 e^{j\omega t}}$



WHAT CURRENT IN LOAD ?

R: $I = \frac{V}{R} = \frac{V_0}{R} e^{j\omega t}$

C: $I = C \frac{dV}{dt} = j\omega C V_0 e^{j\omega t}$

L: $I = \frac{1}{L} \int V dt = \frac{1}{j\omega L} V_0 e^{j\omega t}$

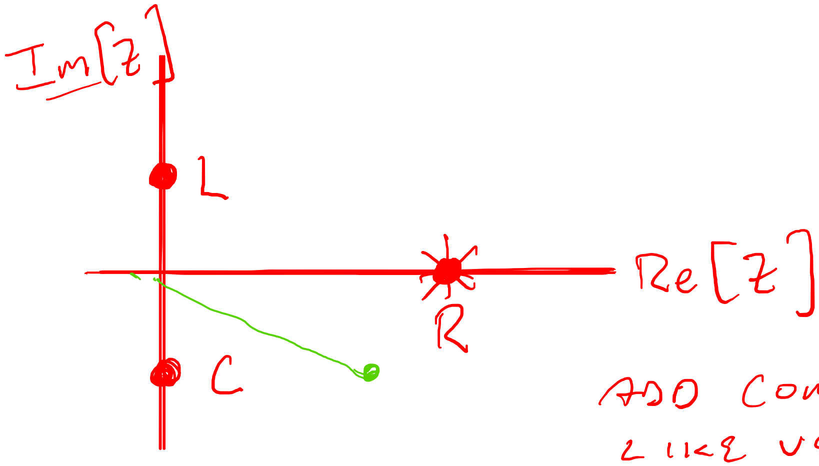
DEFINE IMPEDANCE $\boxed{Z = \frac{V}{I}}$ GEN'LIZED OHM'S LAW.

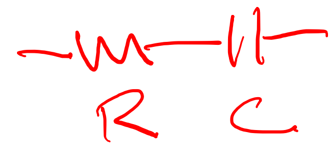
→ R: $Z_R(\omega) = \boxed{R}$

→ C: $Z_C(\omega) = \boxed{1/j\omega C} = -\frac{j}{\omega C}$ VOLTAGE LAGS CURRENT

→ L: $Z_L(\omega) = \boxed{j\omega L}$ VOLTAGE LEAD CURRENT

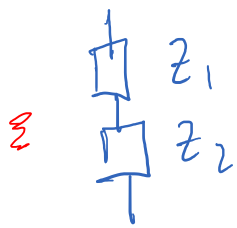
COMPLEX IMPEDANCE PLANE





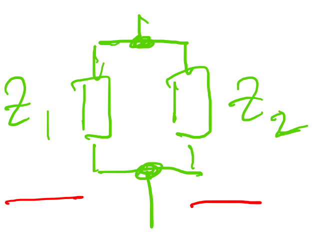
$$\boxed{Z = R - \frac{j}{\omega C}}$$

ADD COMPLEX IMPEDANCES LIKE VECTORS



$$Z_{eff} = Z_1 + Z_2$$

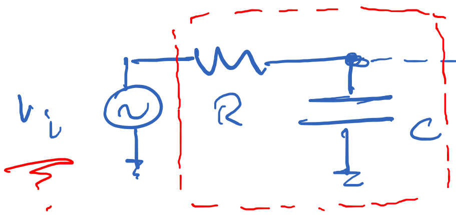
SOMETIMES CONVENIENT TO WORK w/ ADMITTANCE $Y \equiv \frac{1}{Z}$



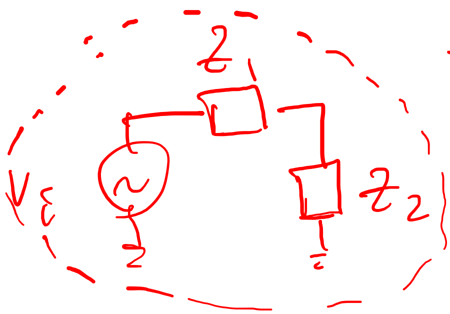
$$Z_{eff} = \left[\frac{1}{Z_1} + \frac{1}{Z_2}\right]^{-1} ; \quad Y_{eff} = Y_1 + Y_2$$

PHASOR EXAMPLE

$V_i e^{j\omega t}$

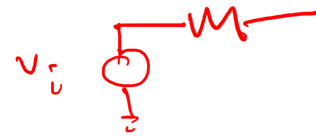


V_c ?



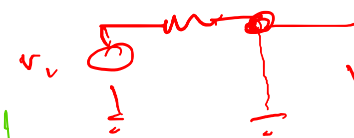
$$\tau = RC$$

LOW FREQ. :



$V_c \to V_i$

HIGH FREQ.



$V_c \to 0$

ASIDE

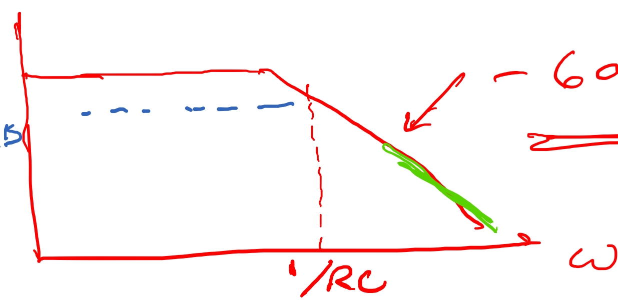
$$\tau = L/R$$

$$V_o = V_i \frac{Z_1}{Z_1 + Z_2}$$

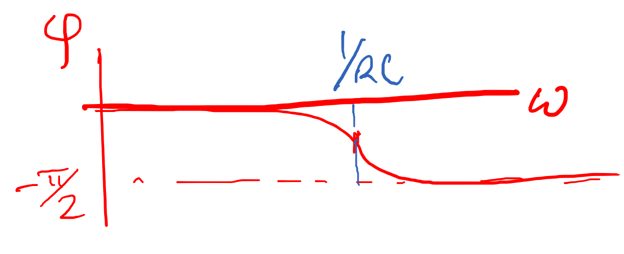
$$V_c = \frac{-j/\omega C}{R - j/\omega C} V_i = \frac{1}{1 + j\omega RC} V_i = \frac{1}{1 + j\omega\tau} V_i$$

$$\left|\frac{V_c}{V_i}\right|^2 = \frac{1}{1+\omega^2\tau^2}$$

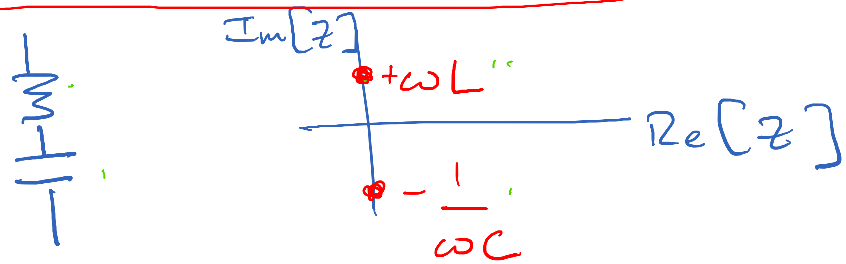
$-3dB$



$-6dB$/OCTAVE



$\boxed{\text{SERIES RLC CIRCUIT}}$



$$Z_{TOT} = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$@\ \omega_0 \equiv \frac{1}{\sqrt{LC}},\quad Z = 0$$

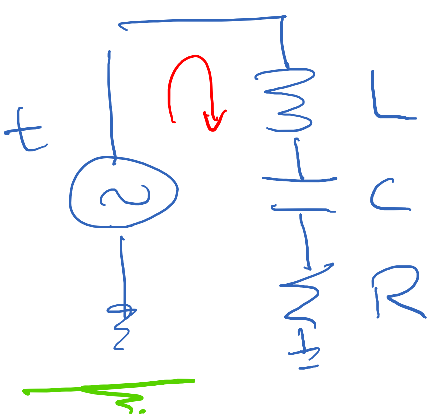
$$\boxed{X \equiv \omega L - \frac{1}{\omega C}}$$

$$I = \frac{V}{R + jX} = \frac{R - jX}{R^2 + X^2}\, V$$

$$|I|^2 = \frac{V_0^2}{R^2 + X^2}$$

CONSIDER

$V_0 e^{j\omega t}$



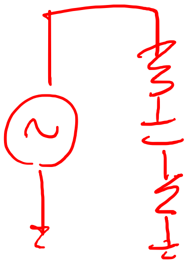
$$\omega_0 L = \frac{1}{\omega_0 C} \equiv Z_0$$

$$Z_0 \gg R$$

$$Z_0;\quad Q \equiv \frac{Z_0}{R} \gg 1$$

$$X = \frac{Z_0}{\omega_0 \omega} \underbrace{(\omega + \omega_0)}_{2\omega_0} \underbrace{(\omega - \omega_0)}_{\Delta}$$

$$X \approx 2 Z_0 \, \Delta / \omega_0$$

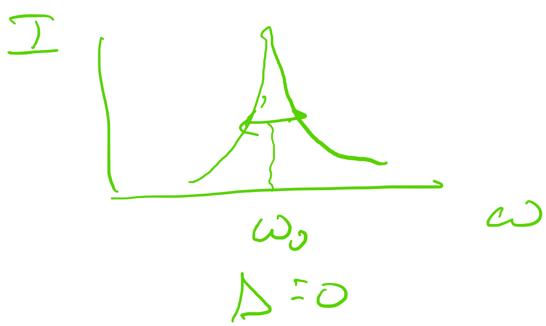


$$|I|^2 = \frac{V_0^2}{\left[\frac{Z_0}{Q}\right]^2 + 4\left[\frac{Z_0}{\omega_0}\right]^2 \Delta^2}$$

$$|I|^2 = \frac{V_0^2}{Z_0^2}\, Q^2 \,\frac{\omega_0^2}{\omega_0^2 + 4Q^2\Delta^2}$$

$$\Delta = 0 \rightarrow I_{PEAK} = \frac{V_0}{Z_0}\, Q$$

CURRENT ENHANCEMENT BY FACTOR Q



$$\text{FWHM} \rightarrow 2\Delta = \omega_0 / Q$$