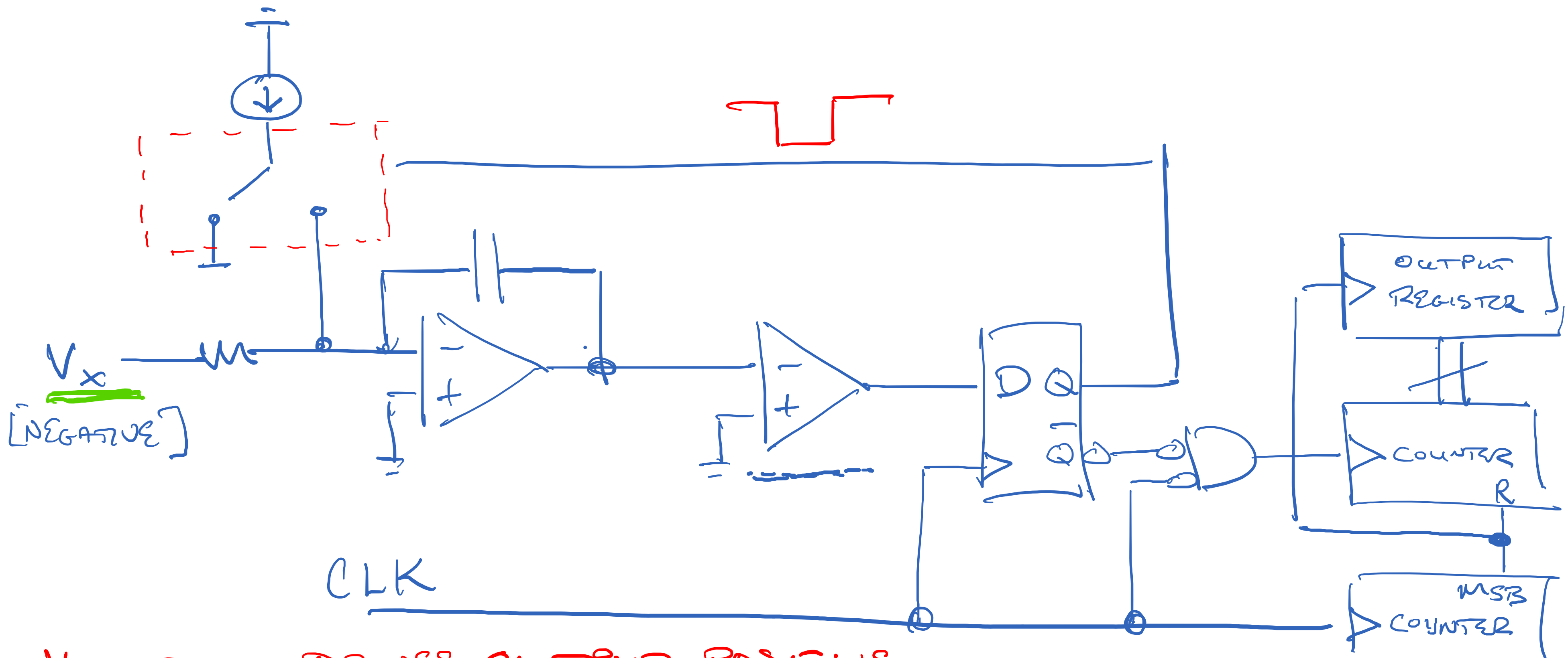
CHARGE-BALANCING ADC : Δ-Σ

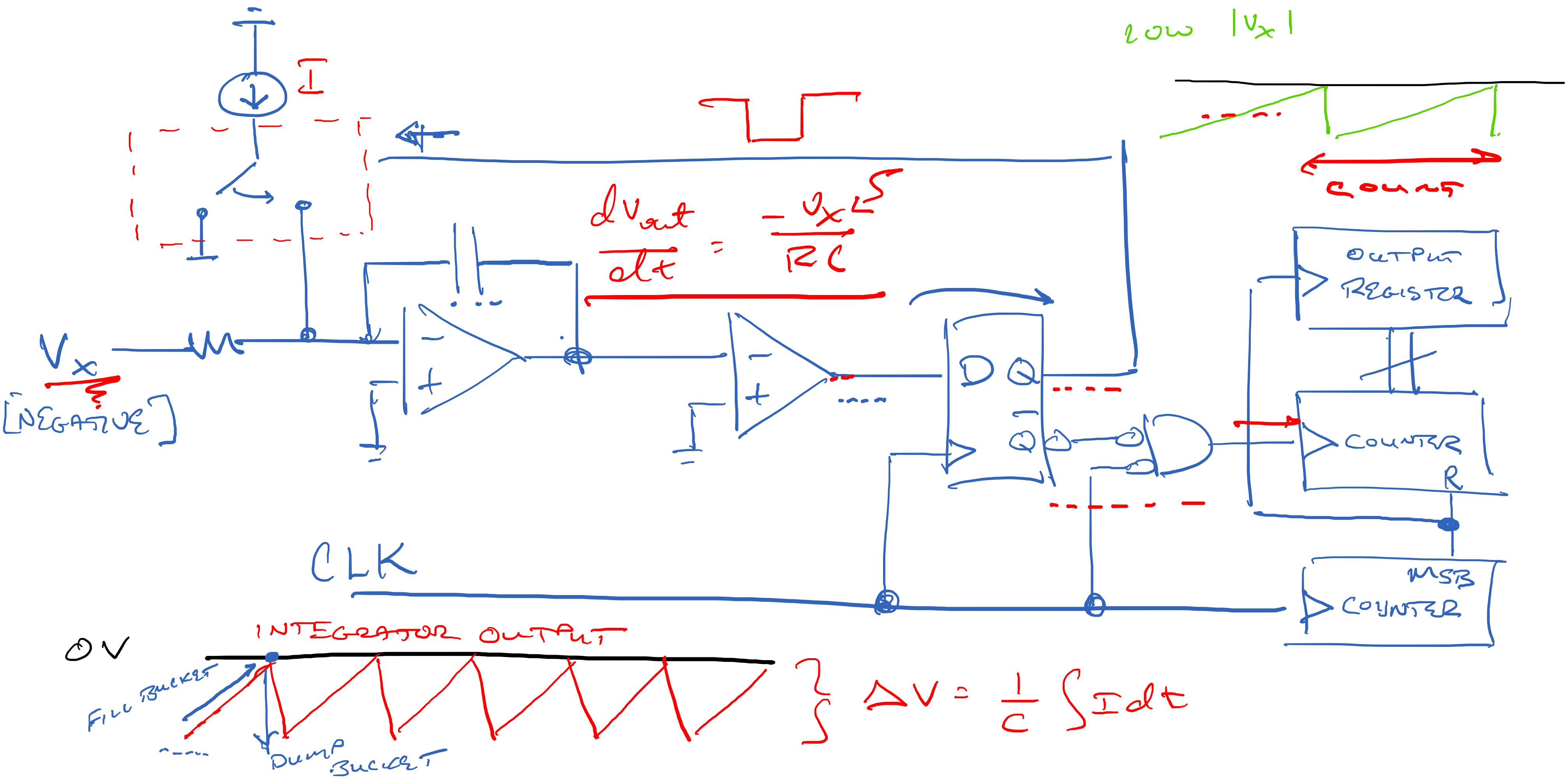


$V_x < 0 \longrightarrow$ DRIVES OUTPUT POSITIVE

DRIVES COMPARATOR NEGATIVE

THIS CAUSES $Q \longrightarrow 0$ [INJECT CHARGE BALANCING PULSE]

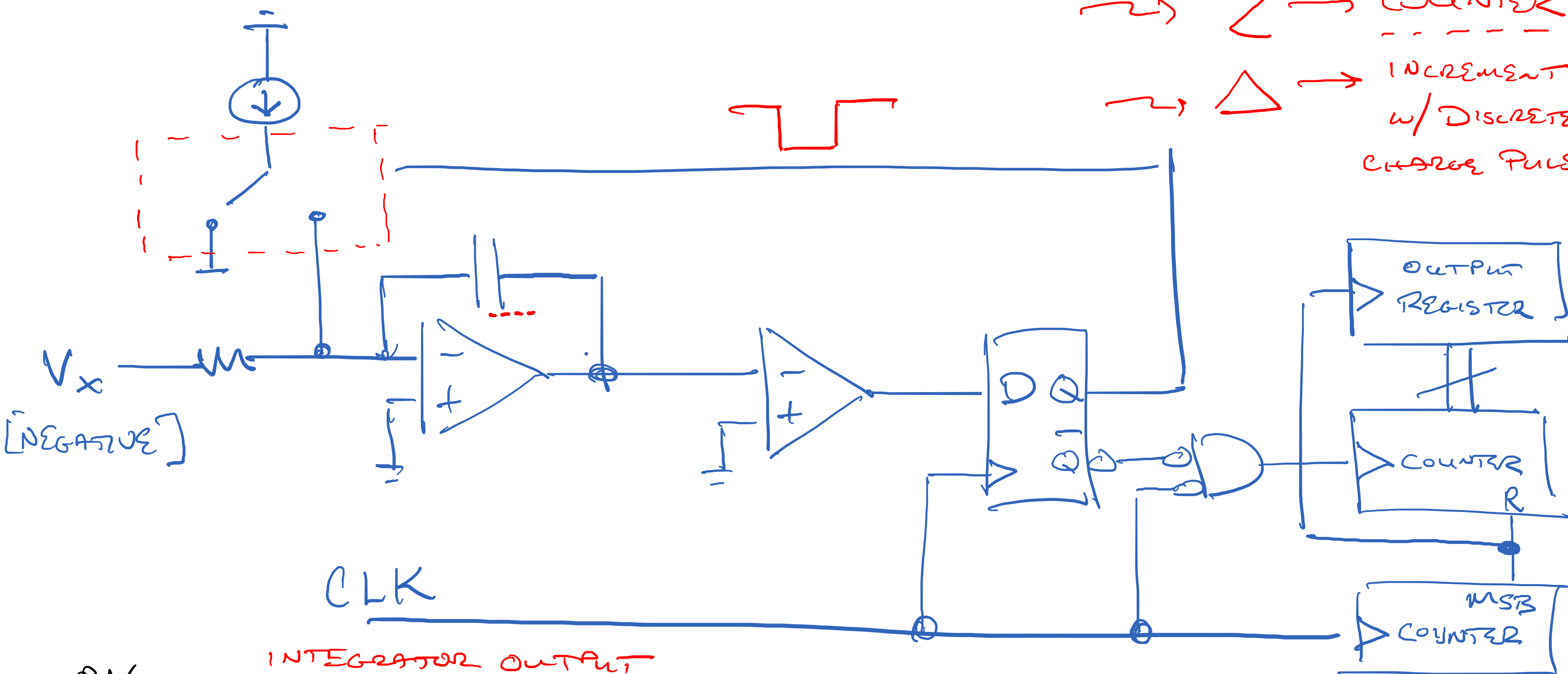
CHARGE-BALANCING ADC : Δ-Σ

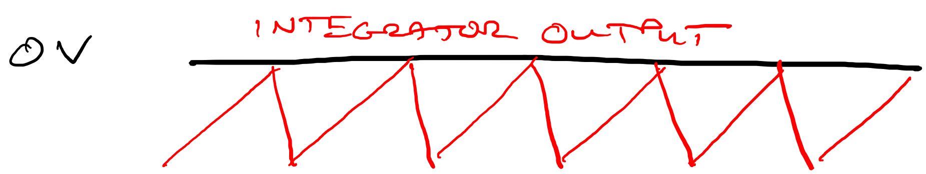


CHARGE-BALANCING ADC: Δ-Σ

→ Σ → COUNTER

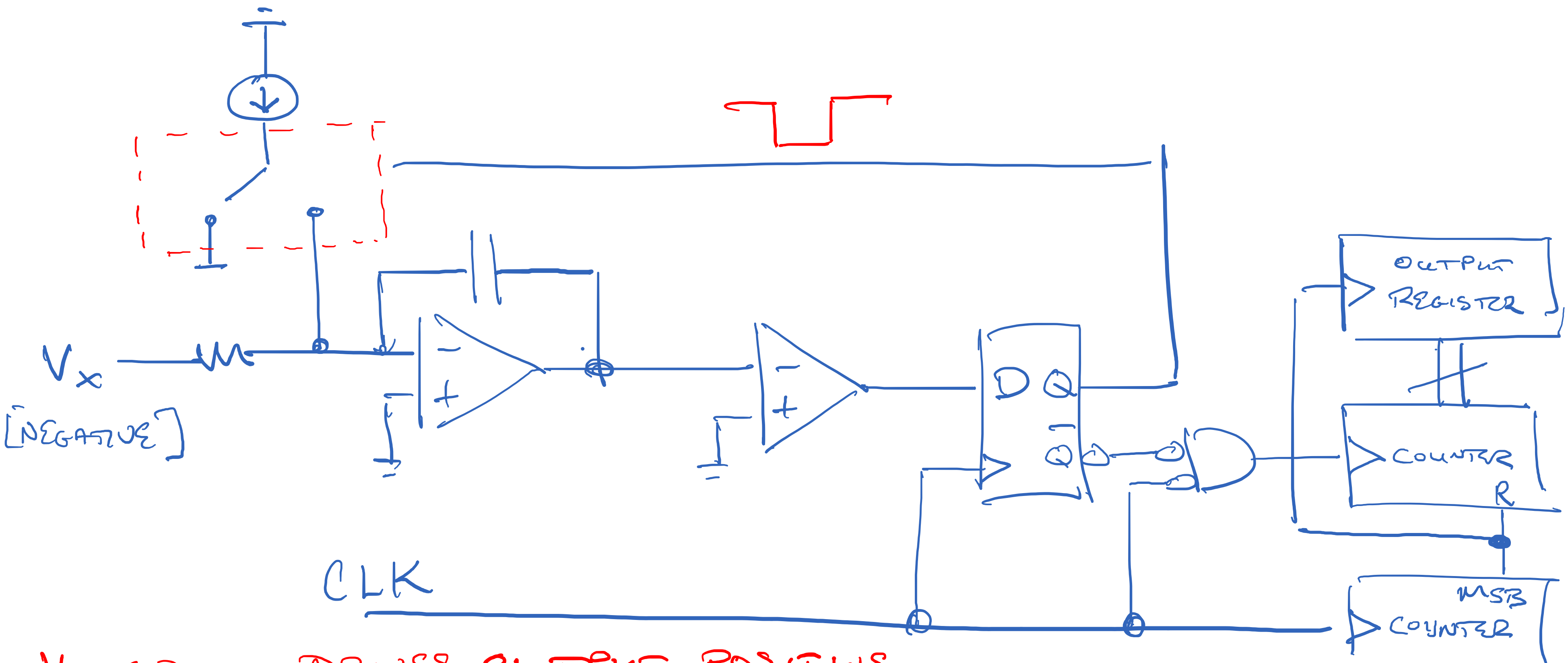
→ Δ → INCREMENT w/ DISCRETE CHARGE PULSES





COUNT CURRENT PULSES REQUIRED TO MAINTAIN COMPARATOR INPUT = 0

CHARGE-BALANCING ADC : Δ-Σ



$V_x < 0$ ⟶ DRIVES OUTPUT POSITIVE

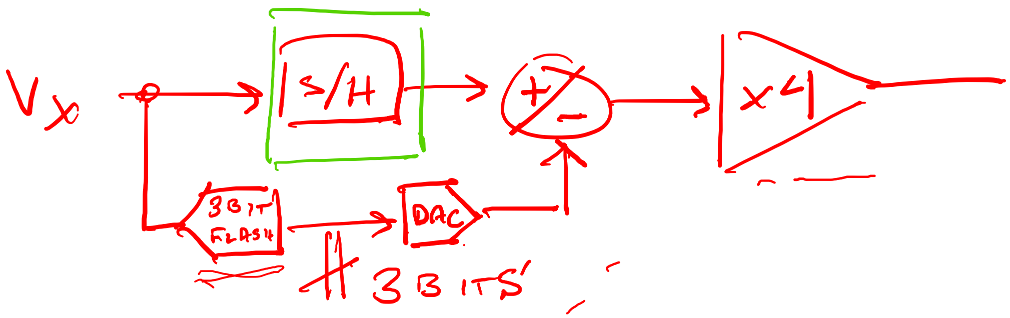
DRIVES COMPARATOR NEGATIVE

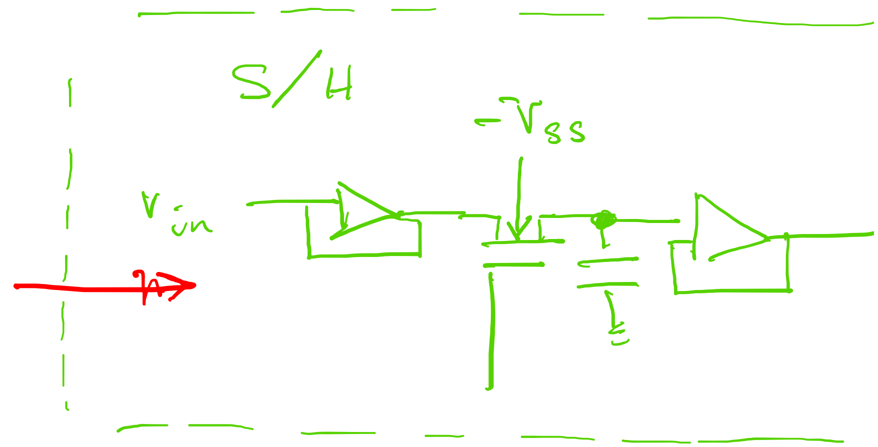
THIS CAUSES Q ⟶ 0 [INJECT CHARGE BALANCING PULSE]

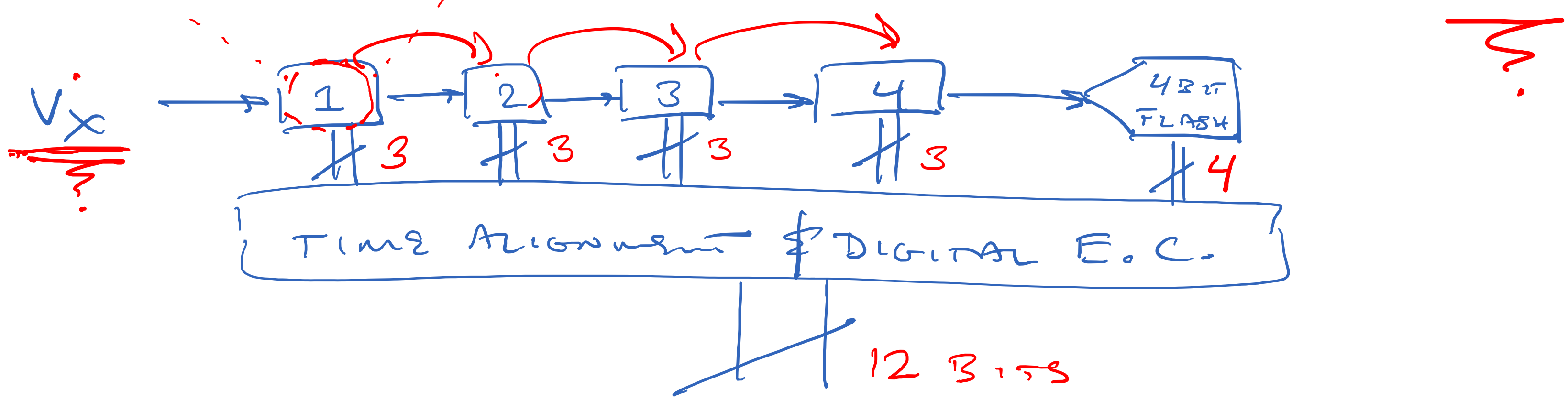
SUMMARY

	SPEED	RESOLUTION
FLASH	FAST [~20 ns]	POOR [→ ~10 BIT]
SINGLE/DUAL SLOPE	SLOW [~ ms]	EXCELLENT [~24 BIT]
SAR	MEDIUM [~ μs]	MODERATE [~16-20 BITS]
$\Delta\Sigma$	MEDIUM [1-10 μs]	MODERATE [~16-20 BITS]

PIPELINED ADC







SEE, e.g., maximintegrated.com

CIRCUIT SIMULATION

SPICE [BERKELEY EECS', 1970's]

SIMULATION PROGRAM WITH INTEGRATED CIRCUIT EMPHASIS

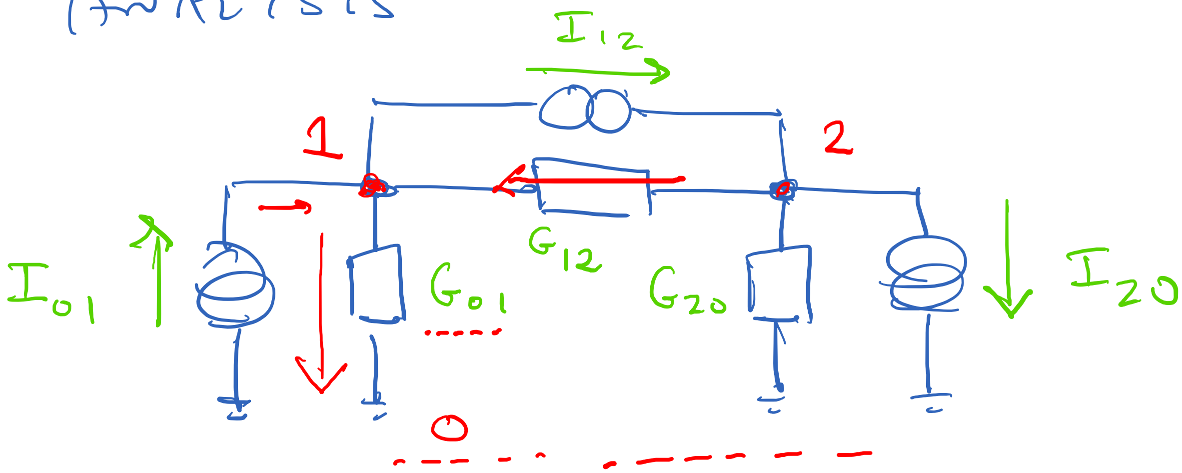
DC ANALYSIS

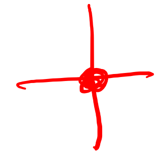
AC ANALYSIS

→ TRANSIENT ANALYSIS

MODELING OF MULTITERMINAL, NONLINEAR DEVICES, ETC.

NODAL ANALYSIS





$$\Sigma I = 0$$

KIRCHHOFF CURRENT LAW:

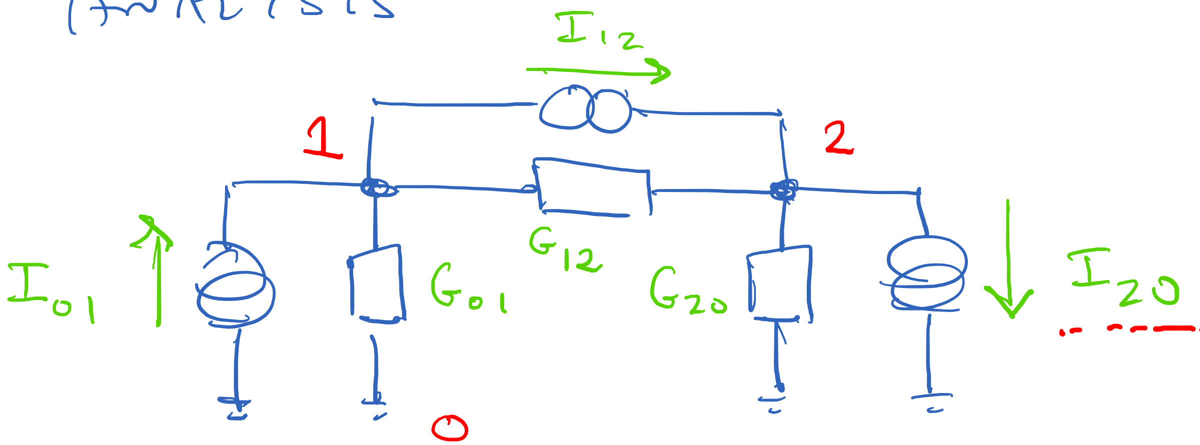
Σ Σ

NODE 0 $\quad G_{01}(V_1 - V_0) - G_{20}(V_0 - V_2) + I_{20} - I_{01} = 0$

NODE 1 $\rightarrow \quad G_{12}(V_2 - V_1) - G_{01}(V_1 - V_0) + I_{01} - I_{12} = 0$

NODE 2 $\quad G_{20}(V_0 - V_2) - G_{12}(V_2 - V_1) + I_{12} - I_{20} = 0$

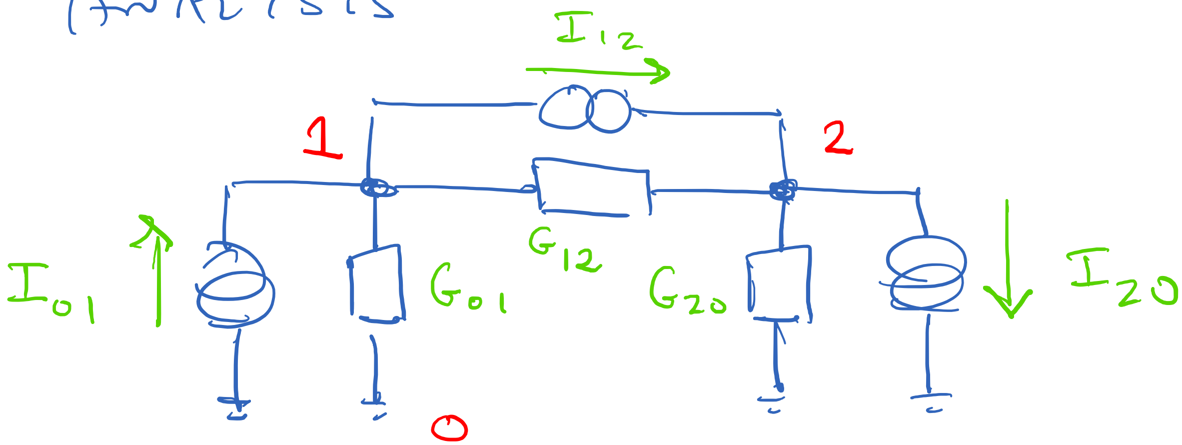
NODAL ANALYSIS



$$\begin{bmatrix} G_{01}+G_{20} & -G_{01} & -G_{20} \\ -G_{01} & G_{12}+G_{01} & -G_{12} \\ -G_{20} & -G_{12} & G_{20}+G_{12} \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{20}-I_{01} \\ I_{01}-I_{12} \\ I_{12}-I_{20} \end{bmatrix}$$

$$\overline{\overline{G}} \cdot \overline{V} = \overline{I}$$

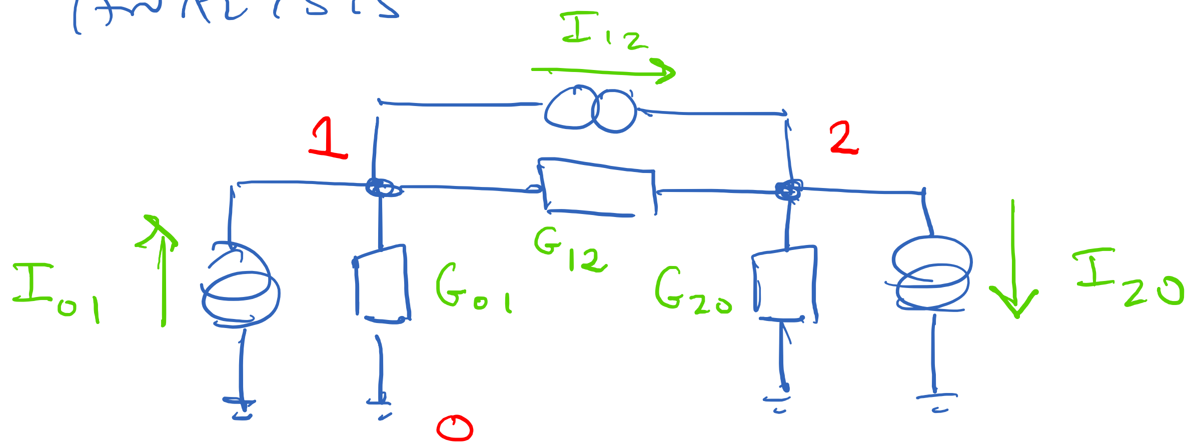
NODAL ANALYSIS



$$\longrightarrow \quad \underline{\underline{\bar{\bar{G}}}} \; \underline{\bar{V}} = \underline{\bar{\bar{I}}}$$

$$\longrightarrow \quad \underbrace{\bar{V}}_{\text{SOLUTION}} = \underline{\bar{\bar{G}}}^{-1} \; \underline{\bar{\bar{I}}}$$

NODAL ANALYSIS

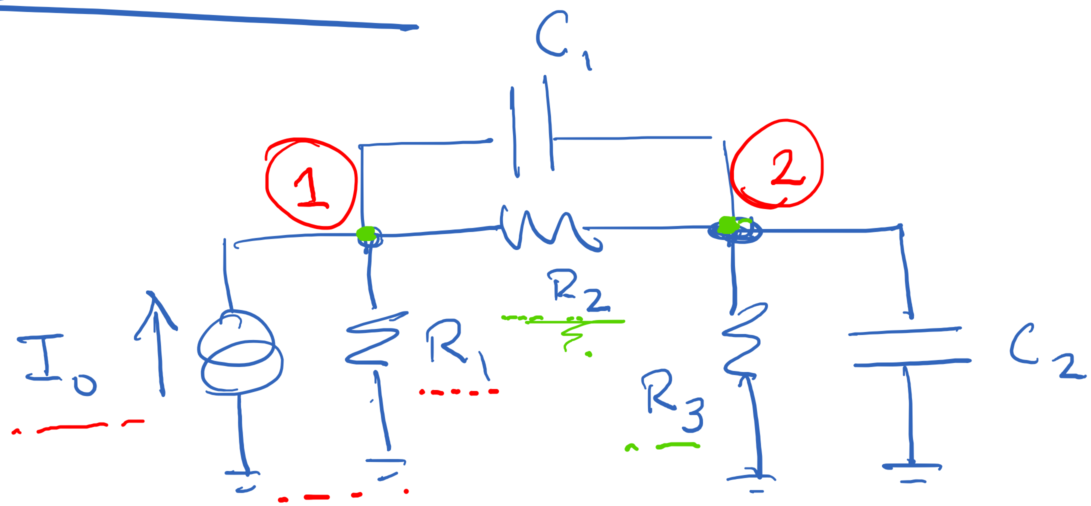


"PATTERN OF FOUR"

$$
\begin{array}{c|c}
 & \begin{array}{ccc} x & & y \end{array} \\
\begin{array}{c} \\ x \\ \\ y \end{array} & \begin{bmatrix} \ddots & & & \\ & +G_{xy} & \cdots & -G_{xy} \\ & \vdots & & \vdots \\ & -G_{xy} & \cdots & +G_{xy} \\ & & & \ddots \end{bmatrix}
\end{array}
$$

CONCRETE EXAMPLE

DC ANALYSIS ONLY

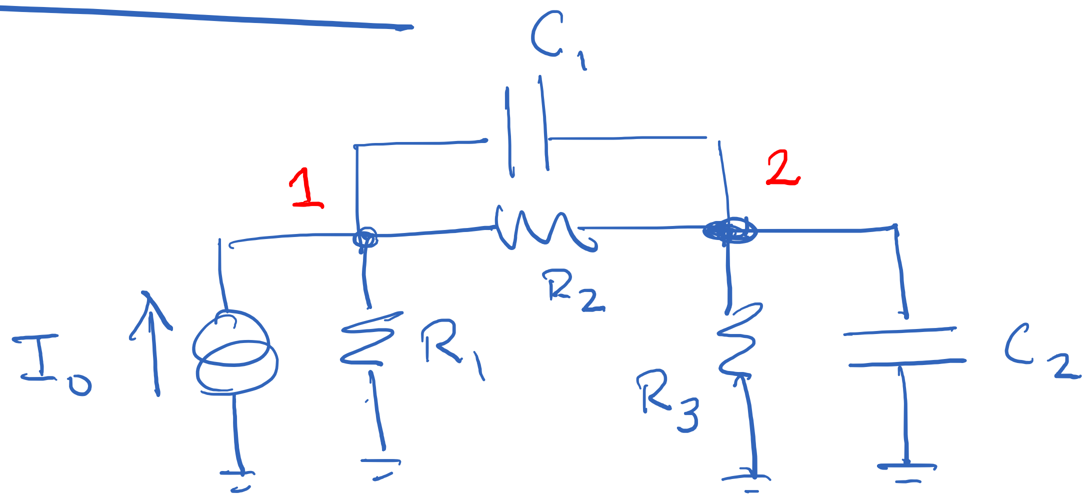


$$
\begin{array}{c} \\ 1 \\ \\ 2 \end{array}
\begin{array}{c}
\begin{array}{cc} \quad 1 \quad & \quad\quad 2 \end{array} \\
\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}
\end{array}
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \end{bmatrix}
$$

KNOWN

KNOWN

CONCRETE EXAMPLE

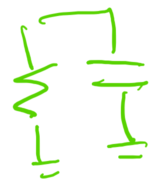


INVERT :

$$\bar{V} = \bar{\bar{G}}^{-1} \bar{I}$$

Extension to AC circuits

$$\bar{V} = \bar{\bar{C}}^{-1}\bar{Q}$$

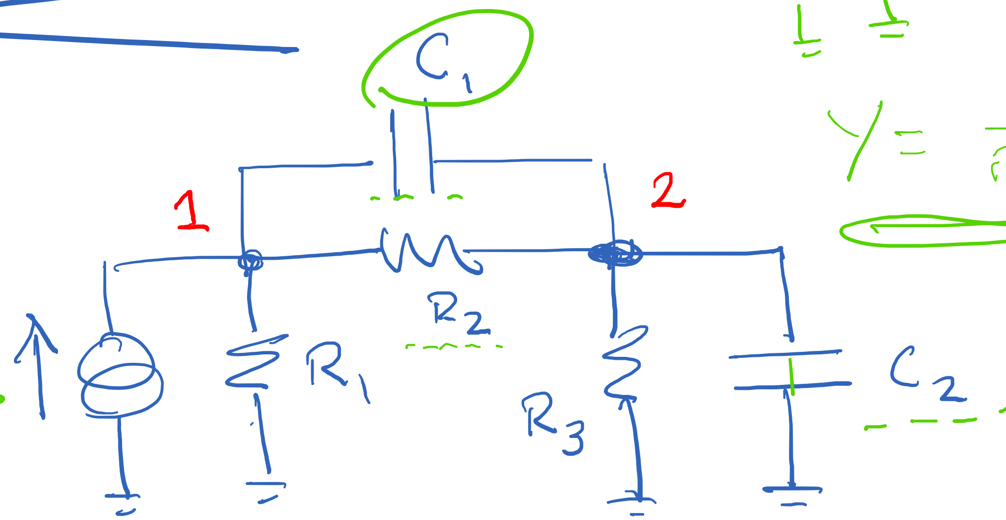


$$Y = \frac{1}{R} + j\omega C$$

$$\bar{\bar{Y}}\,\bar{V} = \bar{I}$$

$$\bar{V} = \bar{\bar{Y}}^{-1}\bar{I}$$

$$I_0 e^{j[\omega t + \varphi]}$$



$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + j\omega C_1 & -\frac{1}{R_2} - j\omega C_1 \\ -\frac{1}{R_2} - j\omega C_1 & \frac{1}{R_3} + \frac{1}{R_2} + j\omega (C_1 + C_2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_0 e^{j\varphi} \\ 0 \end{bmatrix}$$

NONLINEAR CIRCUITS

→ DC ANALYSIS :

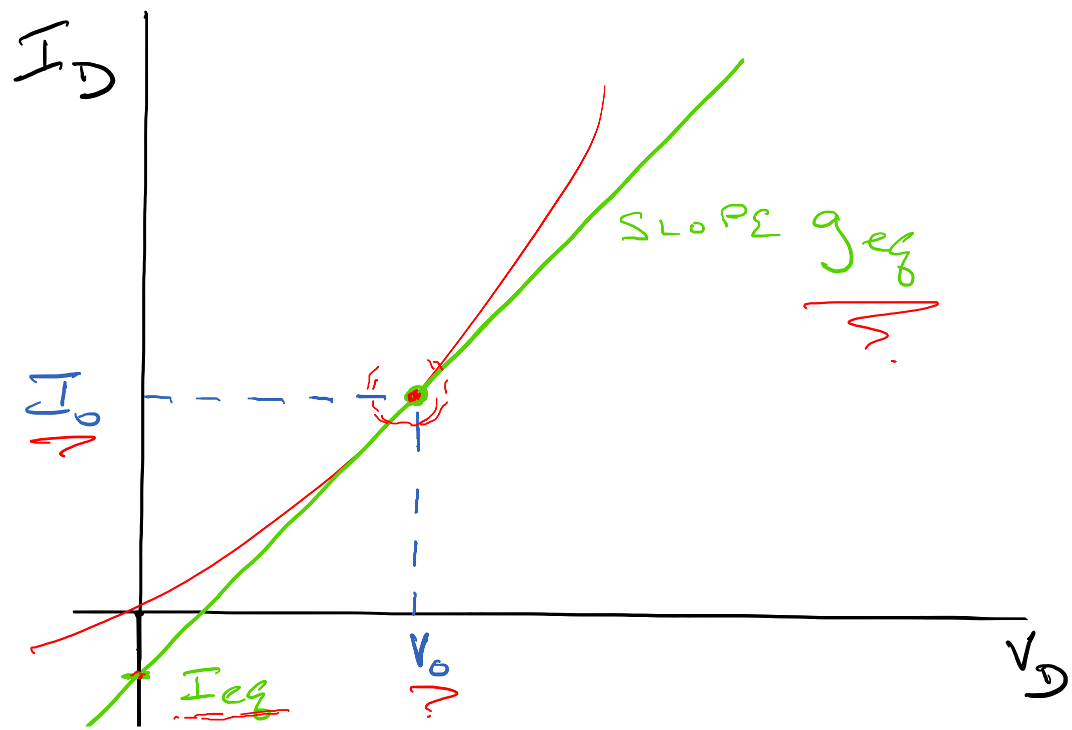
DC SOLUTION OBTAINED BY ITERATIVE PROCEDURE

DIODE

$$I_D = I_s\left[e^{V_D/V_T} - 1\right]$$

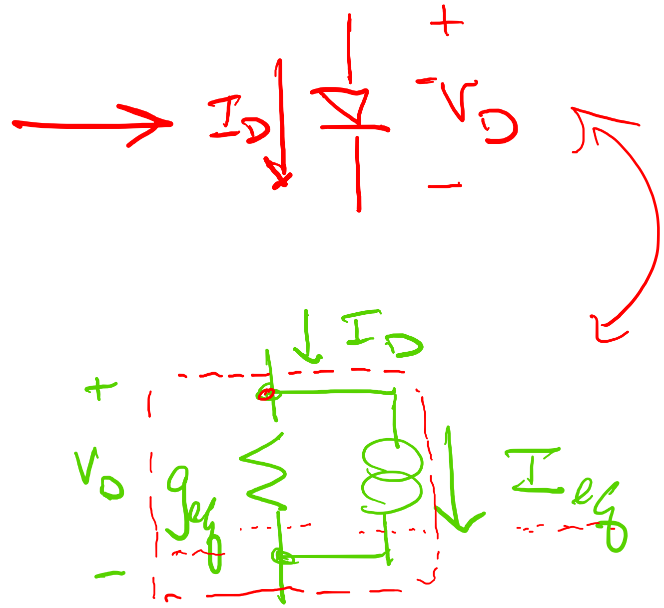
$$g_{eq} = \frac{I_S}{V_T} e^{V_D/V_T}$$

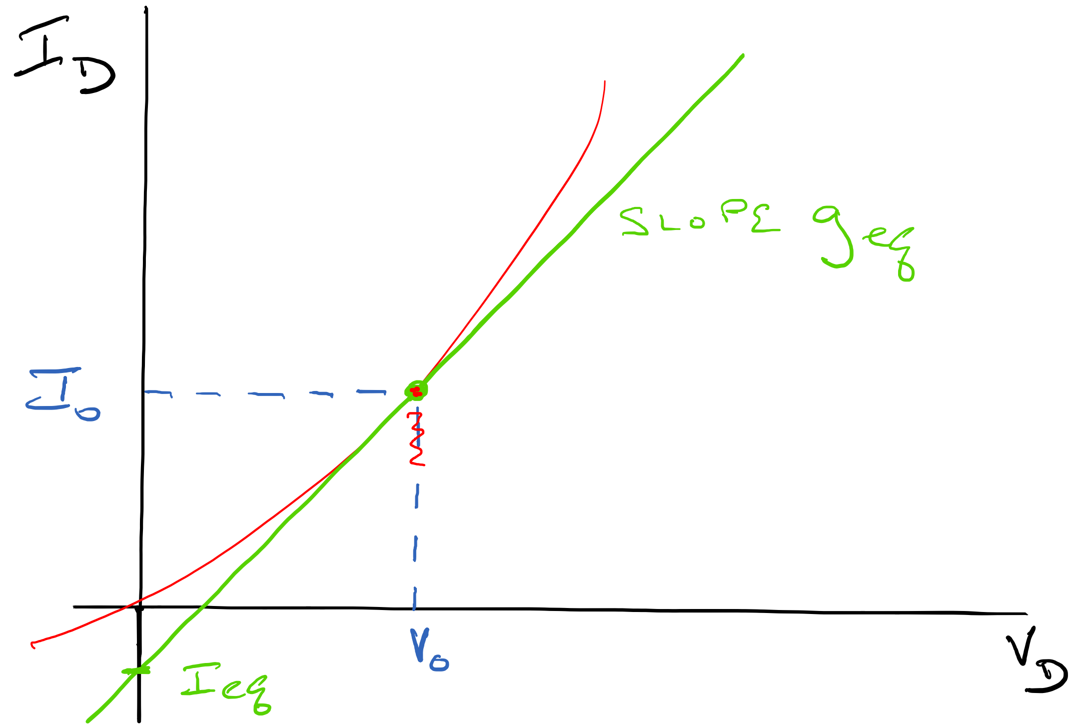
$$I_{eq} = I_O - g_{eq} V_O$$



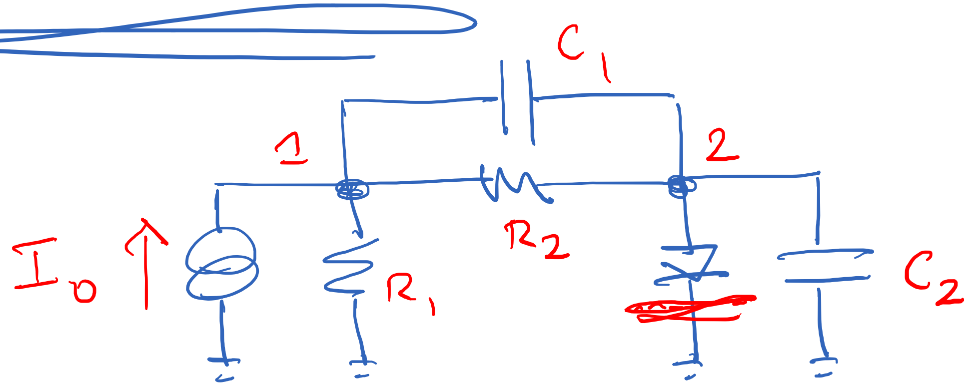
$$g_{eq} = \frac{I_S}{V_T} e^{V_D/V_T}$$

$$I_{eq} = I_D - g_{eq} V_D$$

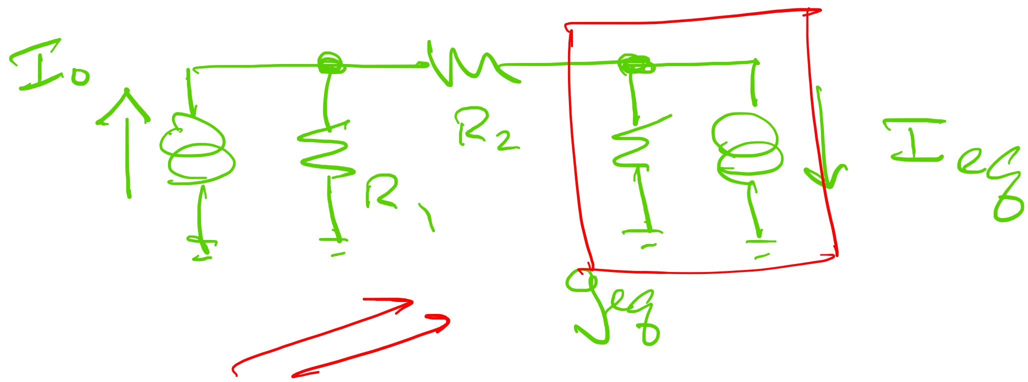




CONCRETE EXAMPLE

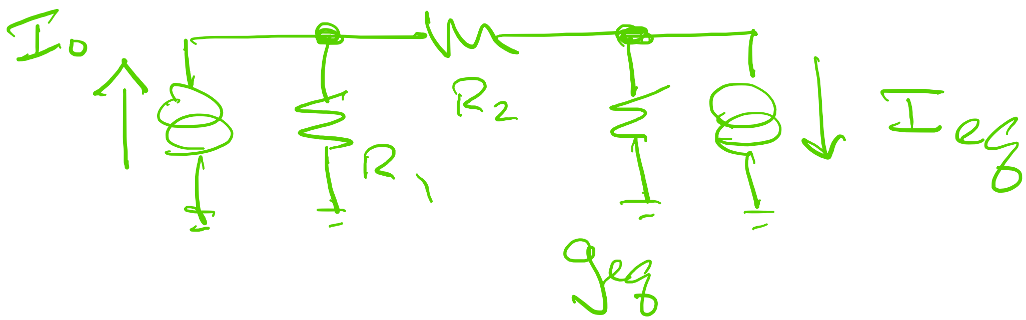


DC
LINEARIZED
EQUIVALENT



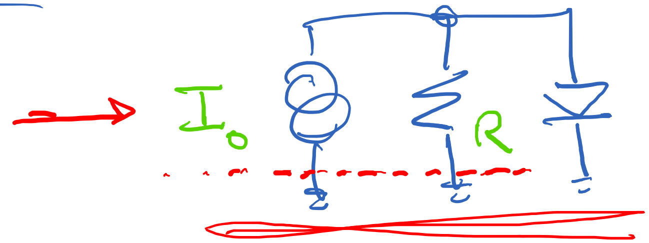
CONCRETE EXAMPLE

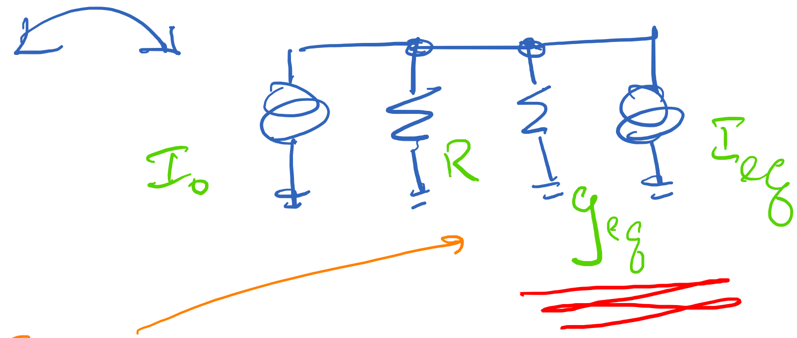
DC
LINEARIZED
EQUIVALENT

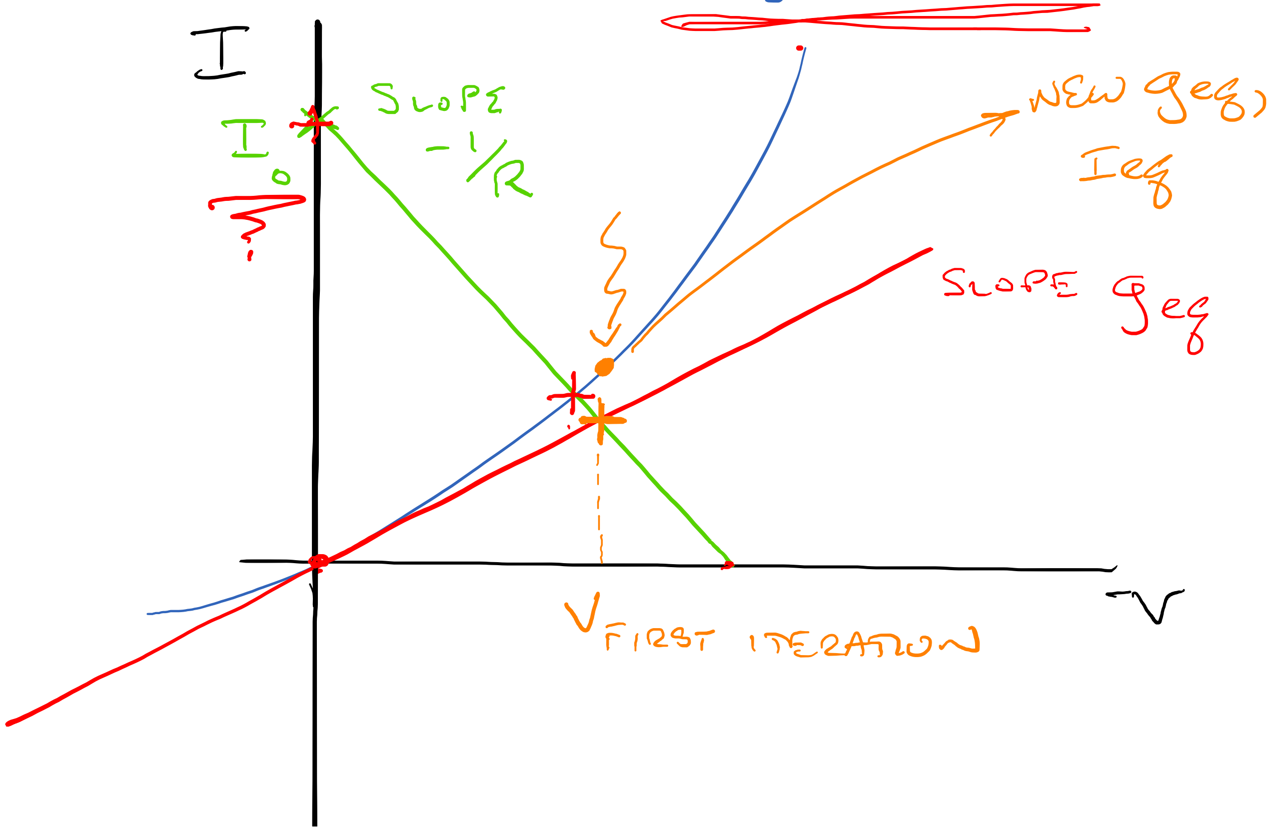


$$\begin{bmatrix} 1/R_1 + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + g_{eq} \end{bmatrix} \begin{bmatrix} V_1 \\ V_D \end{bmatrix} = \begin{bmatrix} I_0 \\ -I_{eq} \end{bmatrix}$$

EVEN SIMPLER



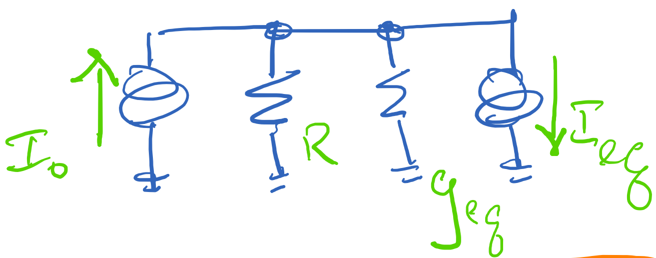




FIRST ITERATION:

$$g_{eq}(V=0) = \frac{I_S}{V_T}\cancel{\exp(0)}$$

$$I_{eq} = 0$$

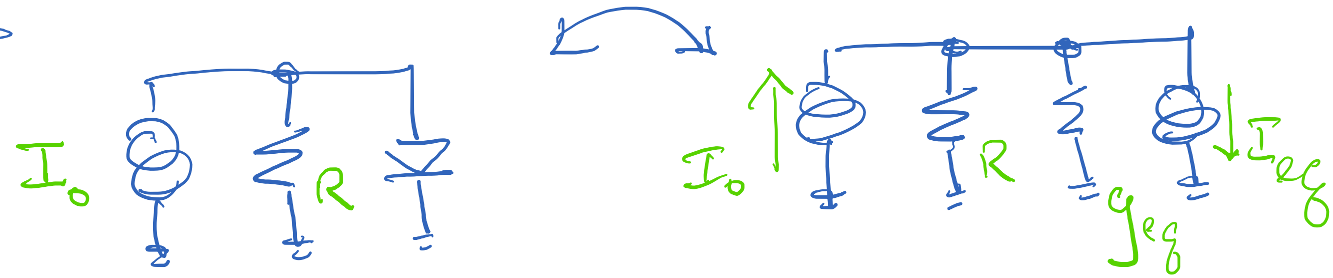


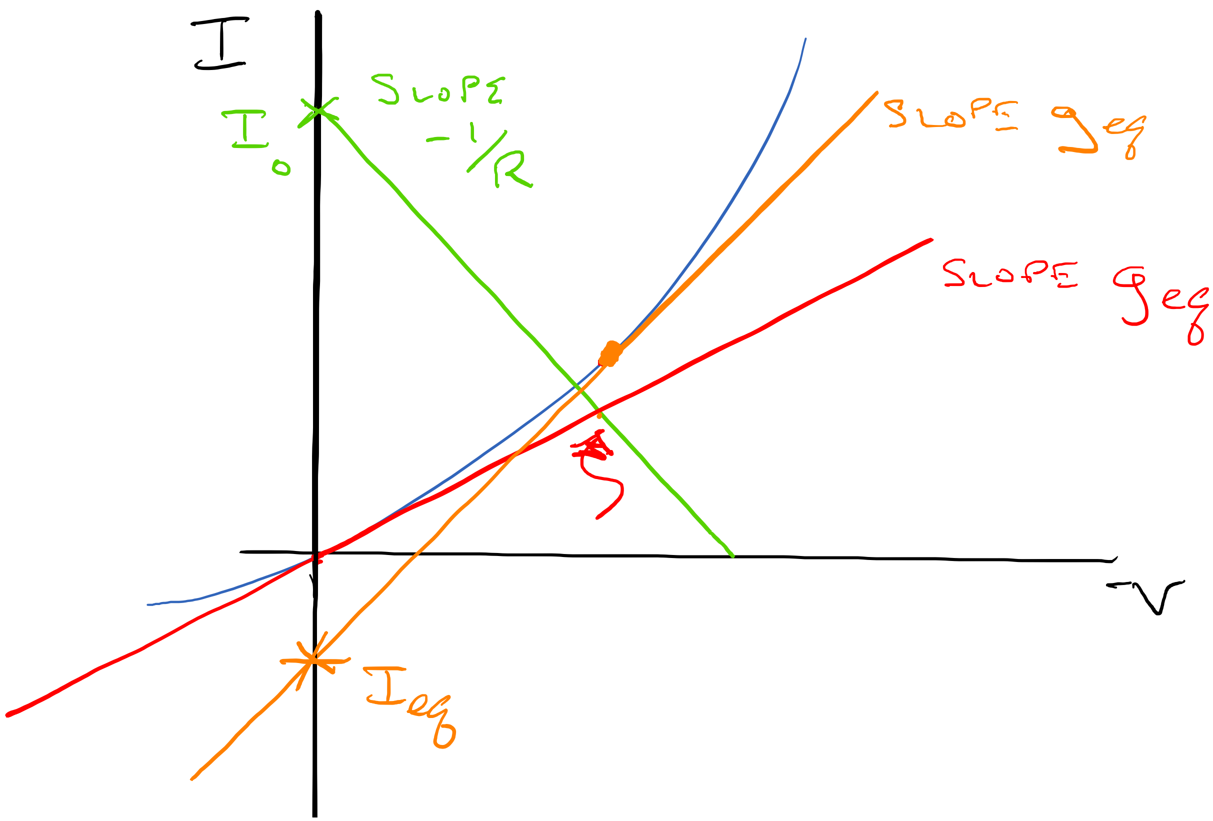
DC ANALYSIS :

$$\underbrace{\left[\frac{1}{R} + g_{eq}\right]}_{} \underset{?}{V} = I_o - I_{eq}$$

$$V = \frac{I_o - I_{eq}}{\frac{1}{R} + g_{eq}} \implies \text{ITERATE}$$

EVEN SIMPLER





UPDATE g_{eq}, I_{eq}

$$V = \frac{I_0 - I_{eq}}{\frac{1}{R} + g_{eq}}$$

ETC.

ETC.

TRANSIENT ANALYSIS

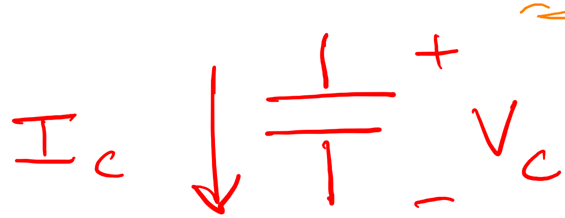
DISCRETIZE TIME INTERVAL $(0, T) \rightarrow (0, t_1, t_2, \ldots, T)$

NUMERICAL INTEGRATION:

$$\rightarrow x_{n+1} = x_n + \Delta t \, \dot{x}_{n+1}$$

@ EACH TIMESTEP, USE ITERATIVE SOLUTION METHOD DESCRIBED ABOVE FOR NONLINEAR DC ANALYSIS

EXAMPLE: NONLINEAR CAPACITOR



$$\rightarrow Q_c = f(V_c)$$

$$\rightarrow I_c = \dot{Q}_c$$

$$Q_{n+1} = Q_n + \Delta t \, \dot{Q}_{n+1}$$

$$I_{n+1} = \frac{Q_{n+1}}{\Delta t} - \frac{Q_n}{\Delta t}$$

KNOWN

$$\boxed{I_{n+1} = \frac{f(V_{n+1})}{\Delta t} - \frac{Q_n}{\Delta t}}$$

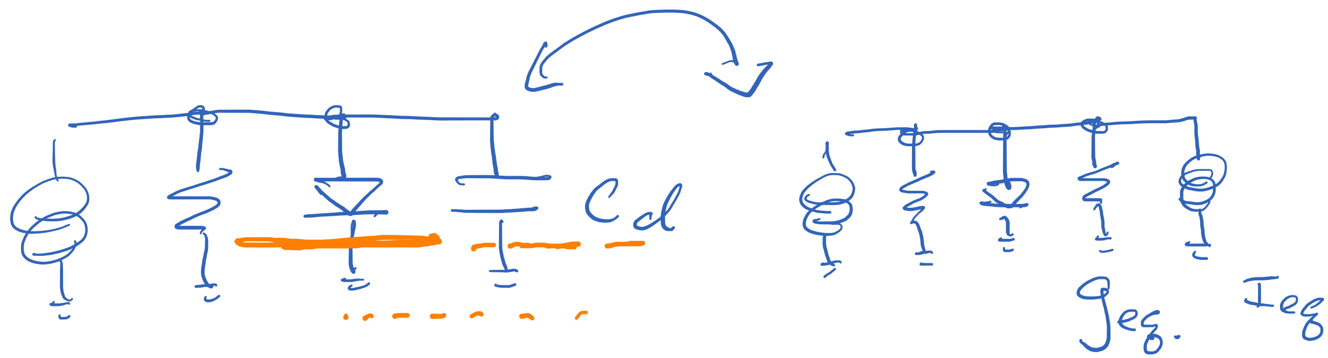
KNOWN FROM SOLUTION @ t_n

→ APPLY NONLINEAR

→ METHOD FOR

DC CIRCUITS.

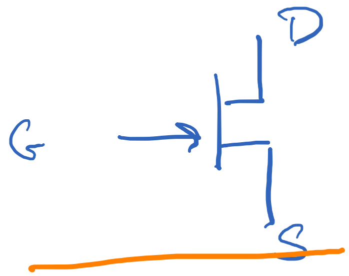
Example

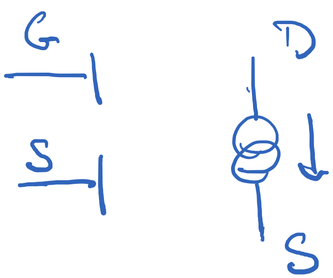


$$\rightarrow C_d = \tau I_s \left[e^{V_c/V_T} - 1 \right]$$

$$\rightarrow I_{n+1} = \frac{\tau I_s}{\Delta t} \left[e^{V_{n+1}/V_T} - 1 \right] - \frac{Q_n}{\Delta t}$$

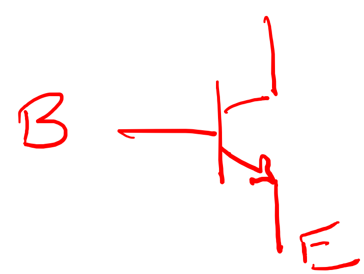
3/4 TERMINAL DEVICES?

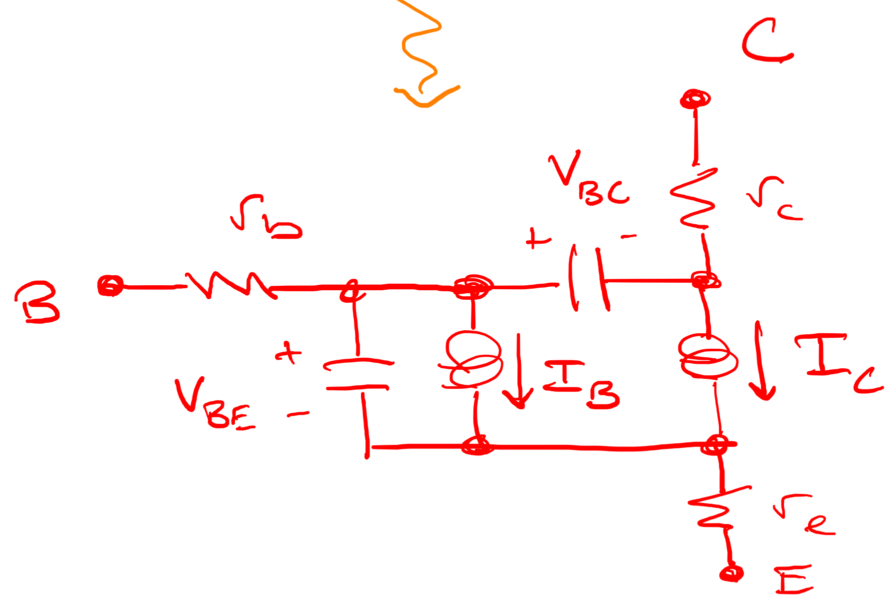




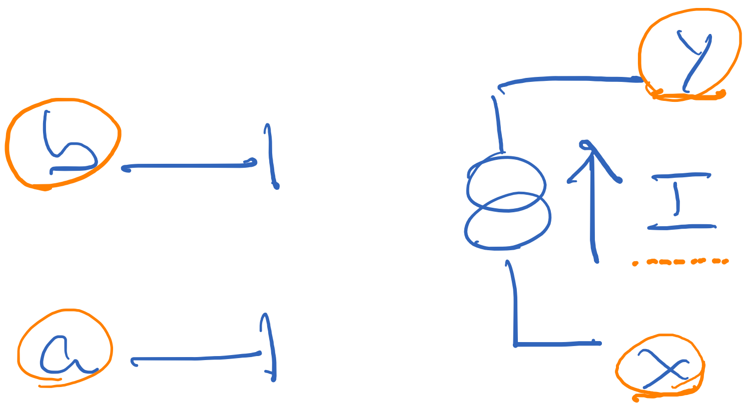
$$I = g\left[V_G - V_S\right]$$

BJT





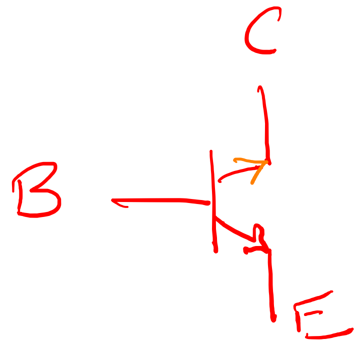
GENERALLY,



$$I = G\,(V_b - V_a)$$

TRANSCONDUCTANCE

$$\bar{\bar{G}} = \begin{array}{c} \\ a \\ b \\ x \\ y \end{array} \begin{array}{c} \begin{array}{cccc} a & b & x & y \end{array} \\ \left[\begin{array}{cccc} \cdot & & \cdot & \\ & \cdot & & \cdot \\ +G & -G & \cdot & \\ -G & +G & & \cdot \end{array}\right] \end{array} \begin{bmatrix} V_a \\ V_b \\ V_x \\ V_y \end{bmatrix}$$

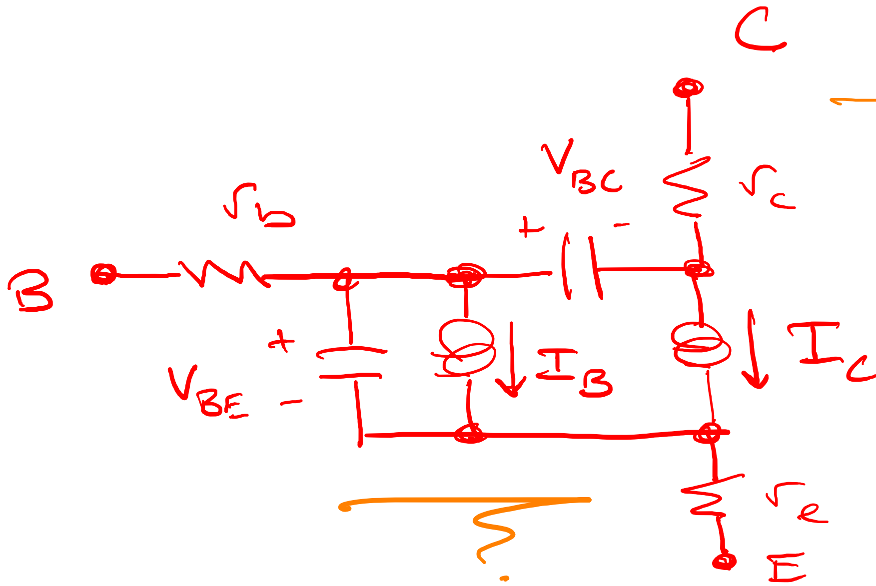


Ebers – Moll

$$I_C = I_S \left[e^{V_{BE}/V_T} - \cancel{e^{V_{BC}/V_T}} \right]$$

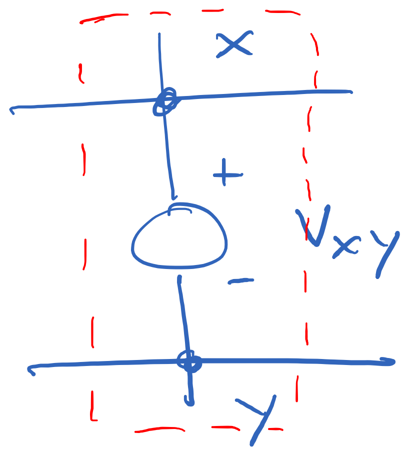
$$I_B = \frac{I_S}{\beta} \left[e^{V_{BE}/V_T} - 1 \right]$$

[Simplified]



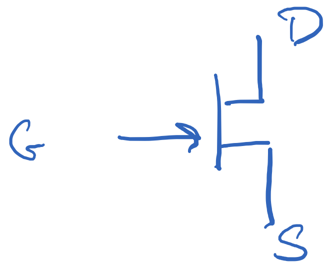
VOLTAGE SOURCES

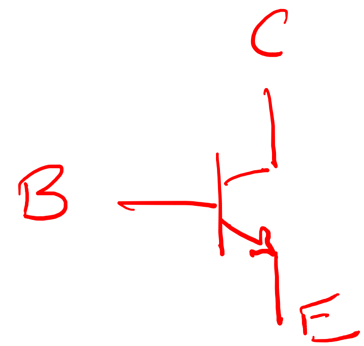
MODIFIED NODAL ANALYSIS

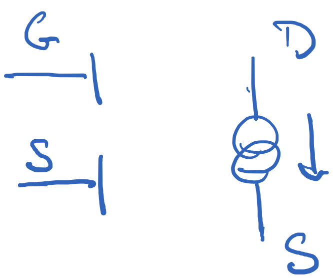


"SUPER - NODE"

3/4 TERMINAL DEVICES ?







$I = g\left[V_G - V_S\right]$