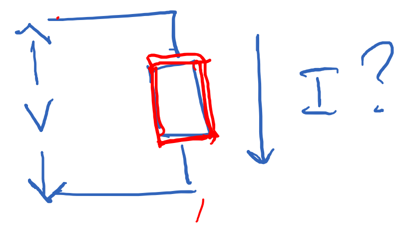
TAKE $\underline{V = V_0 e^{j\omega t}}$



WHAT CURRENT IN LOAD ?

R: $I = \frac{V}{R} = \frac{V_0}{R} e^{j\omega t}$

C: $I = C \frac{dV}{dt} = j\omega C V_0 e^{j\omega t}$

L: $I = \frac{1}{L} \int V dt = \frac{1}{j\omega L} V_0 e^{j\omega t}$

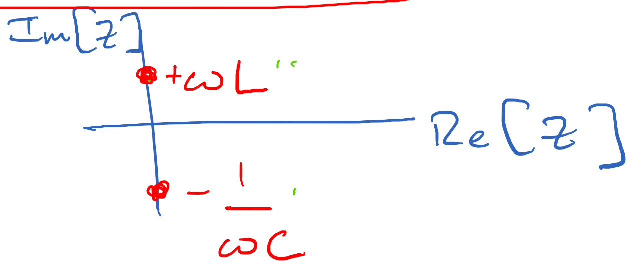
DEFINE IMPEDANCE $\boxed{Z = \frac{V}{I}}$ GEN'LIZED OHM'S LAW.

→ R: $Z_R(\omega) = \boxed{R}$

→ C: $Z_C(\omega) = \boxed{1/j\omega C} = -\frac{j}{\omega C}$ VOLTAGE LAGS CURRENT

→ L: $Z_L(\omega) = \boxed{j\omega L}$ VOLTAGE LEAD CURRENT

SERIES RLC CIRCUIT



$$\boxed{X \equiv \omega L - \frac{1}{\omega C}}$$

$$I = \frac{V}{R + jX} \; ; \; |I|^2 = \frac{V_0^2}{R^2 + X^2}$$

$$Z_{TOT} = j\left(\omega L - \frac{1}{\omega C}\right)$$

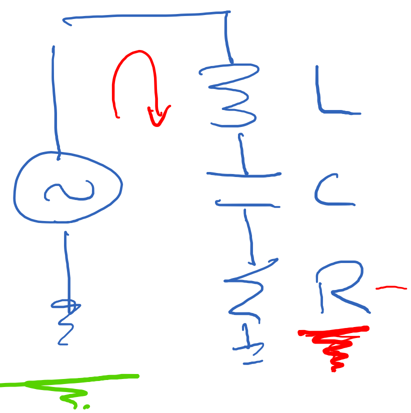
$\rightarrow$ @ $\omega_0 \equiv \frac{1}{\sqrt{LC}}$, $Z = 0$

INTRODUCE $\boxed{Z_0 \equiv \omega_0 L = \frac{1}{\omega_0 C}} = \sqrt{\frac{L}{C}}$

WORK IN LIMIT $Q \equiv \frac{Z_0}{R} >> 1$

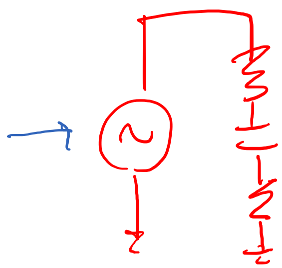
CONSIDER

$\rightarrow$ $V_0 e^{j\omega t}$



$$X = \frac{Z_0}{\omega_0 \omega} \underbrace{(\omega + \omega_0)}_{2\omega_0} \underbrace{(\omega - \omega_0)}_{\Delta}$$

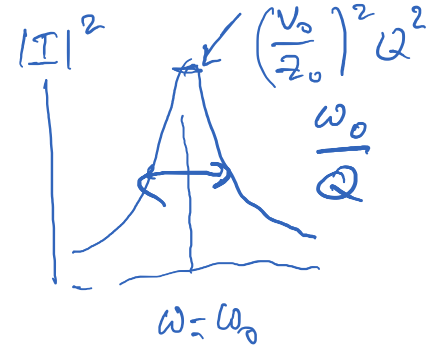
$$X \approx 2 Z_0 \Delta / \omega_0$$



$\omega < \omega_0$

$|I| \sim \frac{V_0}{Z_0}$

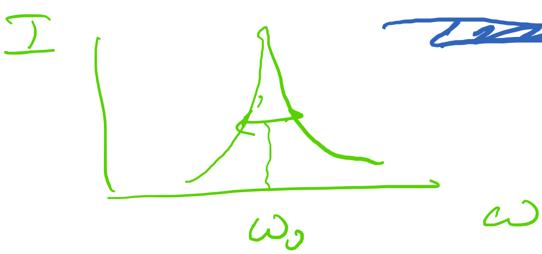
$$\boxed{|I|^2 = \frac{V_0^2}{\left[\frac{Z_0}{Q}\right]^2 + 4\left[\frac{Z_0}{\omega_0}\right]^2 \Delta^2}}$$



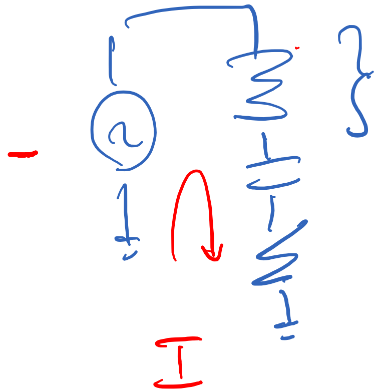
$$|I|^2 = \frac{V_0^2}{Z_0^2} Q^2 \frac{\omega_0^2}{\omega_0^2 + 4Q^2\Delta^2}$$

$$\Delta = 0 \rightarrow I_{PEAK} = \frac{V_0}{Z_0} Q = \frac{V_0}{R}$$

CURRENT ENHANCEMENT BY FACTOR Q

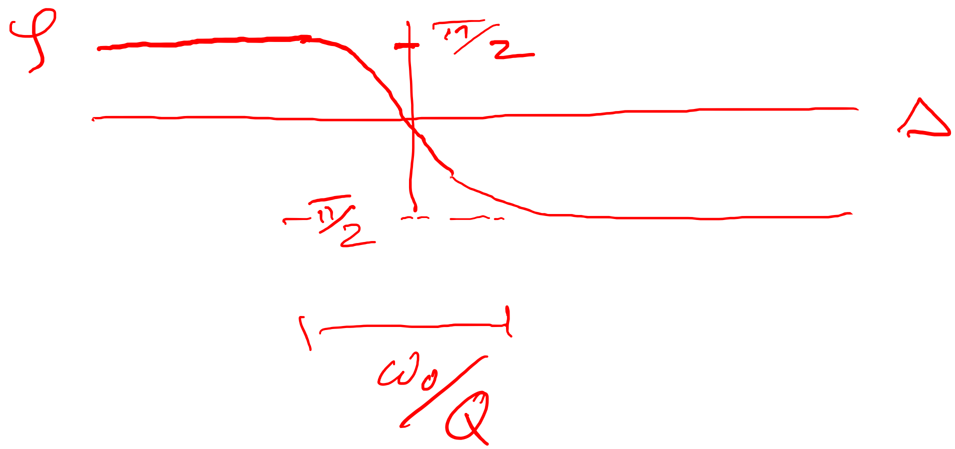


FWHM $\rightarrow 2\Delta = \omega_0 / Q$



$$V_L = \frac{V_0}{\cancel{Z_0}} \cdot Q \times j\cancel{Z_0} = j\underbrace{Q V_0}$$

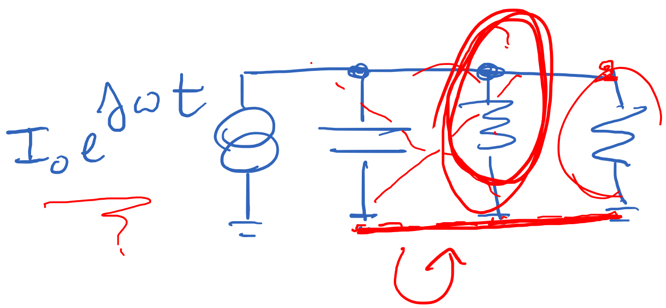
$$V_C = \frac{V_0}{Z_0} \cdot Q \cdot (-jZ_0) = -jQV_0$$



VOLTAGE ENHANCEMENT

NOT AMPLIFICATION!

PARALLEL LC RESONANCE



$$Y(LC) = \frac{1}{j\omega L} + j\omega C$$

$$Y = 0 \ @ \ \omega_0 = \frac{1}{\sqrt{LC}}$$

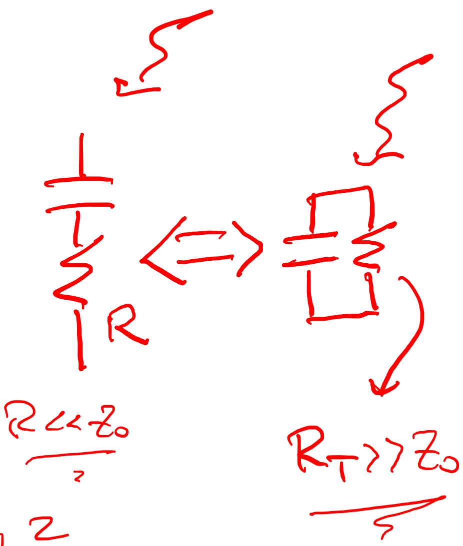
AGAIN, INTRODUCE

$$Z_0 = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{Q = R/\omega_0 L = \omega_0 RC = R/Z_0}$$

$$\Delta = \omega - \omega_0$$

⋮



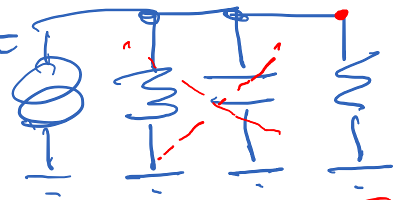
$$|V|^2 = I_0^2 Z_0^2 Q^2 \frac{\omega_0^2}{\omega_0^2 + 4Q^2\Delta^2}$$

ANOTHER DEFINITION OF Q

$$Q = \frac{\omega_0 \times \text{MAX STORED ENERGY}}{\text{POWER DISSIPATION}}$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

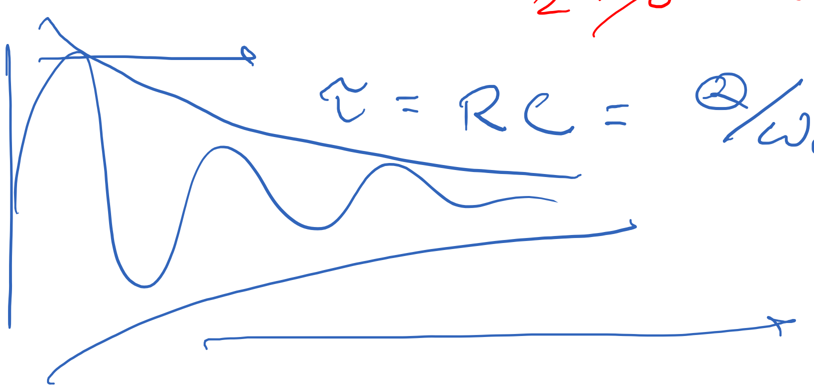
$I_0 e^{j\omega_0 t}$



$V_{max} = I_0 R$

$$Q = \frac{\omega_0 \cdot \cancel{\tfrac{1}{2}} C \cancel{I_0^2} R^2}{\cancel{\tfrac{1}{2}} \cancel{I_0^2} R} = \boxed{\omega_0 RC}$$

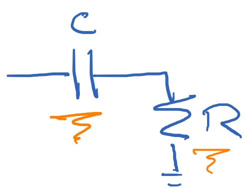
AS EXPECTED



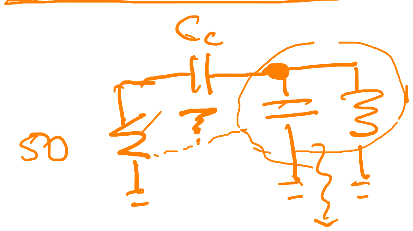
$\tau = RC = Q/\omega_0$

IMPEDANCE TRANSFORMATION

$$X \equiv \frac{1}{\omega C}$$



$$Z = R + \frac{1}{j\omega C} = R - jX$$; TAKE $\boxed{X >> R}$



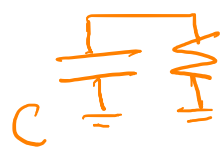
WHAT IS Y TO GROUND?

$$Y = \frac{1}{R - jX} = \frac{R + jX}{R^2 + X^2} \approx \boxed{\frac{R}{X^2} + \frac{j}{X}}$$

$C \sim 1pF$

$\tau = 5 \cdot 10^{-11} \to 50ps$

$\Longrightarrow$



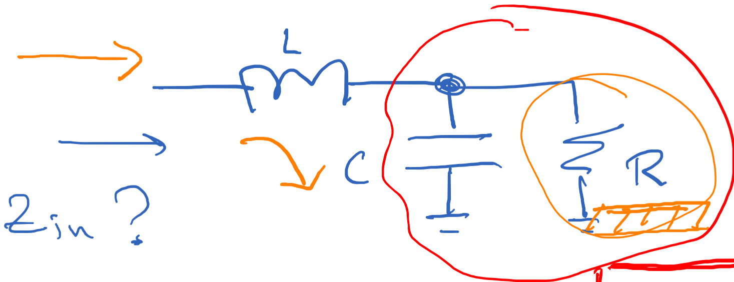
$$R_T = \frac{\left(\frac{1}{\omega C}\right)^2}{R}$$

RESONANT IMPEDANCE TRANSFORMATION

TRANSFORM HIGH RESISTANCE TO LOW RESISTANCE

$$Z_C = \frac{1}{\omega C}$$

$$Z_L = \omega L$$

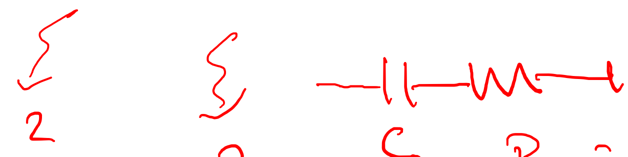


DEFINE $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Z_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

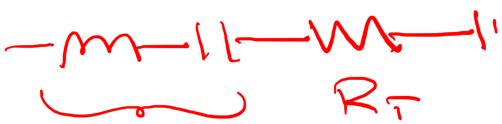
CONSIDER $\boxed{Z_0 \ll R}$

$$Y = \frac{1}{R} + j\frac{1}{Z_C}$$



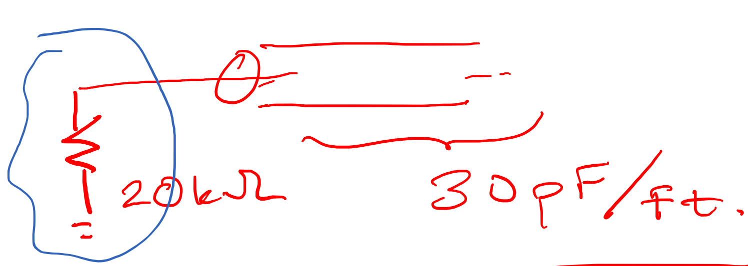
$$Z(RC) = \frac{\frac{1}{R} - j/Z_C}{1/R^2 + \frac{1}{Z_C^2}} \approx \frac{Z_C^2}{R} - jZ_C$$

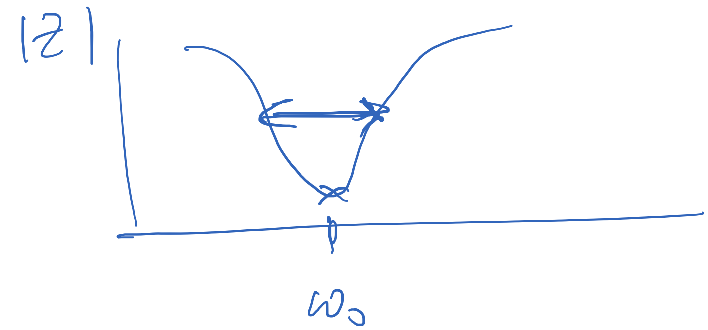
$$R_T = \frac{Z_C^2}{R} \ll R$$



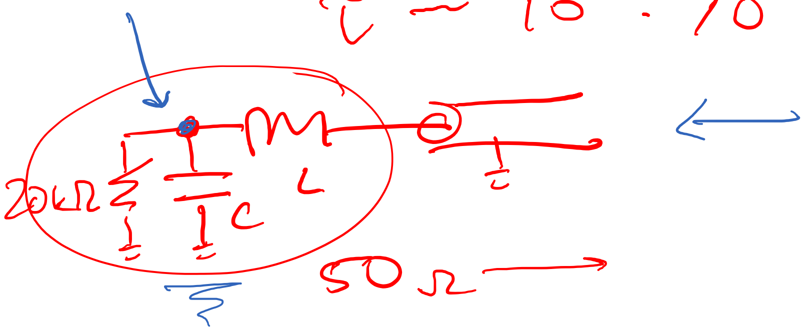
R $\longrightarrow$ TRANSFORMS TO $R_T = \frac{Z_0^2}{R} \ll R$

"RF SET"





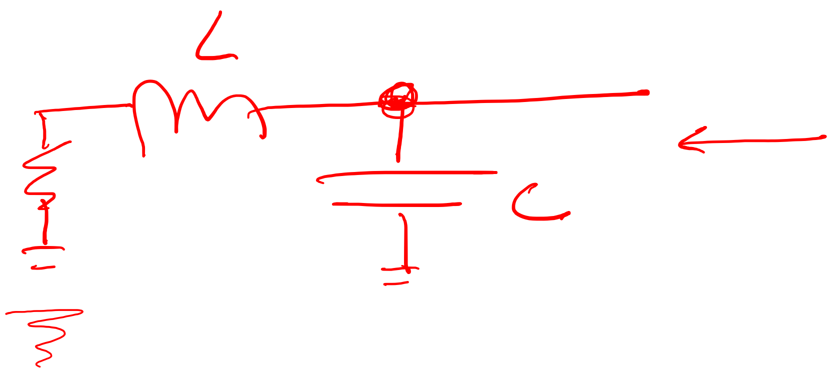
$\tau \sim 10^4 \cdot 10^{-10} \sim 10^{-6}\ s$



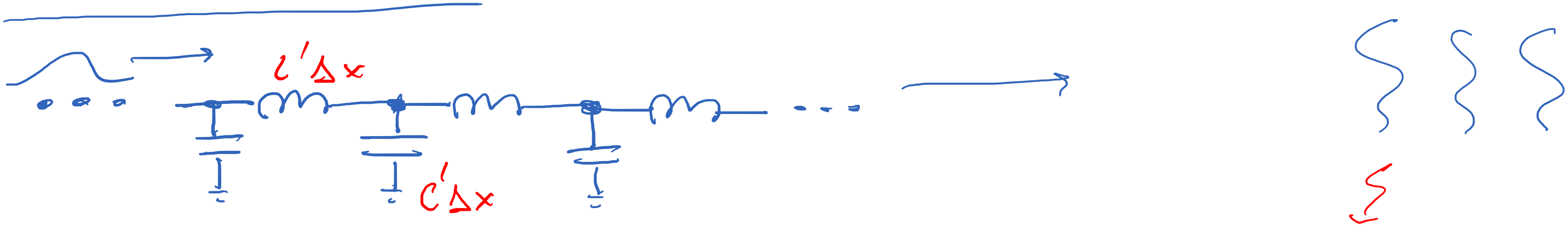
$Q = R/Z_0$

$R_{HIGH} \rightarrow R_{HIGH}/Q^2$

$R \ll Z_0$



TRANSMISSION LINES



KIRCHOFF VOLTAGE LAW: $V(x) - V(x+\Delta x) = L'\Delta x \dfrac{dI}{dt}$

$$\Rightarrow \quad \frac{\partial V}{\partial x} = -L' \frac{\partial I}{\partial t}$$

KIRCHOFF CURRENT LAW: $I(x+\Delta x) = I(x) - C'\Delta x \dfrac{dV}{dt}$

$$\Rightarrow \quad \frac{\partial I}{\partial x} = -C' \frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial x^2} = -L' \frac{\partial^2 I}{\partial x \partial t} \quad ; \quad \frac{\partial^2 V}{\partial t^2} = -\frac{1}{C'} \frac{\partial^2 I}{\partial x \partial t}$$

$$\Longrightarrow \quad \frac{\partial^2 V}{\partial x^2} = L'C' \frac{\partial^2 V}{\partial t^2}$$

, & SIMILARLY FOR I

WAVE EQUATION

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

TRAVELING WAVE SOLUTIONS OF FORM $e^{j(\pm kx - \omega t)} = e^{j\frac{2\pi}{\lambda}(\pm x - vt)}$

WAVELENGTH λ, PHASE VELOCITY v

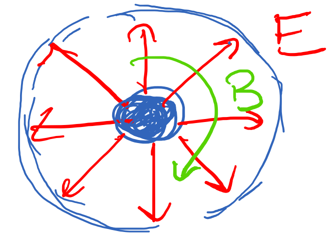
$k = \frac{2\pi}{\lambda}$; $v = \lambda f = \omega / k$

FOR OUR TRANSMISSION LINE, $\frac{\partial^2 V}{\partial x^2} - L'C' \frac{\partial^2 V}{\partial t^2} = 0$

PHASE VELOCITY $v = \frac{1}{\sqrt{L'C'}}$

EXAMPLES

COAXIAL LINE



$$\boxed{C' = \frac{2\pi\epsilon}{\ln(b/a)}} \quad ; \quad \boxed{L' = \frac{\mu}{2\pi}\ln(b/a)}$$

BNC → 30 pF/ft.

$$V = \frac{1}{\sqrt{\epsilon\mu}}$$

$\mu \to \mu_0$

$\epsilon \sim \epsilon_R \epsilon_0$

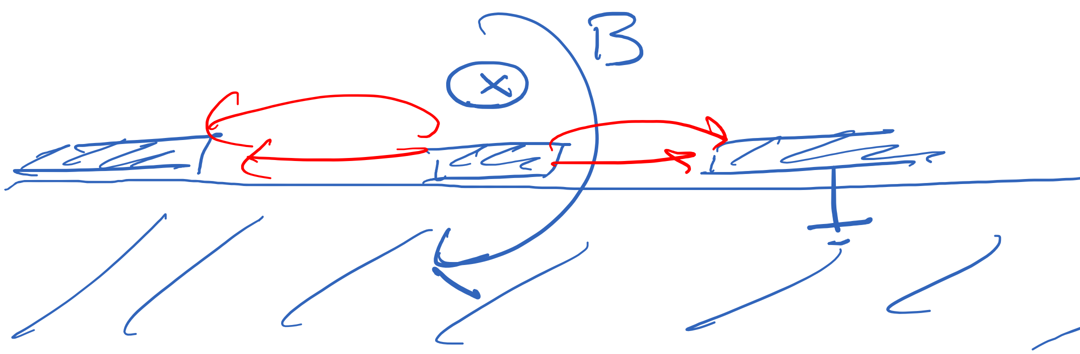
$$V = \frac{c}{\sqrt{\epsilon_R}}$$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

FOR COAX
$V \sim 2 \cdot 10^8$ m/s

[5ns FOR 1m]

COPLANAR WAVEGUIDE



$$\boxed{v(x,t) = V_0 e^{j(kx-\omega t)}}$$

WHAT IS $I(x,t)$?

$$\frac{\partial I}{\partial t} = -\frac{1}{L'}\frac{\partial V}{\partial x} = -\frac{1}{L'} jk V_0 e^{j(kx-\omega t)}$$

$$I = -\frac{1}{L'} jk V_0 \int e^{j(kx-\omega t)} \, dt$$

$$= +\frac{1}{L'}\frac{jk}{j\omega} V_0 e^{j(kx-\omega t)}$$

RECALL $\frac{\omega}{k} = \frac{1}{\sqrt{L'C'}}$

$$= \underline{\sqrt{\frac{C'}{L'}}} \, \underbrace{V_0 e^{j(kx-\omega t)}}$$

DEFINE $\underline{Z_0 \equiv \sqrt{\frac{L'}{C'}}}$ CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINE

TRANSMISSION LINE w/ MATCHED TERMINATION