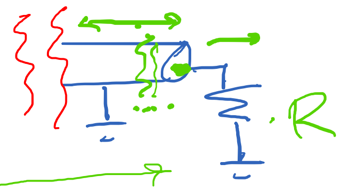
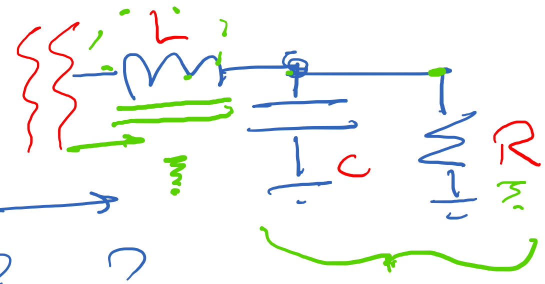
TERMINATED TRANSMISSION LINE

CONSIDER



→



Z_{in}?

$L = L'\Delta x$

$C = C'\Delta x$

LET

$Z_L \equiv \omega L$

$Z_C \equiv 1/\omega C$

$$Z_{in} = jZ_L - \frac{jZ_C R}{R - jZ_C}$$

$$\frac{1}{1+x} \approx 1 - x \qquad x \ll 1$$

→ FIND L, C s.t.

$$R = Z_{in} = jZ_L - \frac{jZ_C R}{R - jZ_C} \approx jZ_L + R\left[1 - j\frac{R}{Z_C}\right]$$

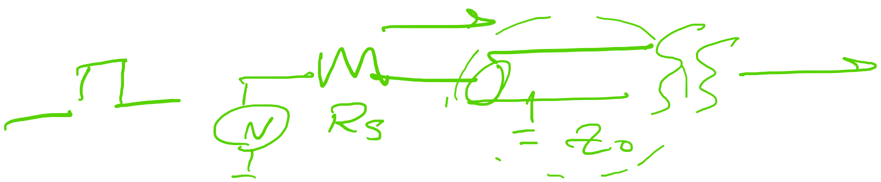
$$\Rightarrow Z_L = \frac{R^2}{Z_C}, \quad \text{OR} \quad R = \sqrt{\frac{L}{C}} = Z_0$$

DEFINE <u>CHARACTERISTIC</u> <u>IMPEDANCE</u>

$$\rightarrow \quad Z_0 = \sqrt{\frac{L'}{C'}}$$

IMPEDANCE OF X'MISSION LINE TERMINATED

$\Rightarrow$ w/ $R = Z_0$ IS <u>PURELY</u> <u>REAL</u>

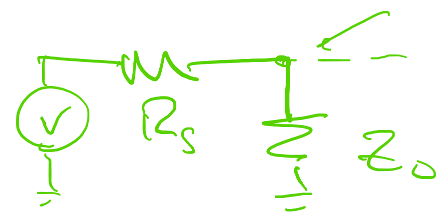


$$\frac{V_i}{i_i} = Z_0$$

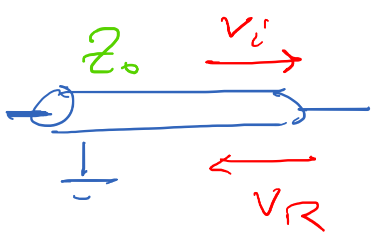
[ALSO, @ SHORT TIMES, IMPEDANCE LOOKING INTO

$\Rightarrow$ TRANSMISSION LINE IS PURELY REAL &

$\Rightarrow$ EQUAL TO Z_0]



THEVENIN EQUIVALENT OF TRANSMISSION LINE



VOLTAGES & CURRENTS ARE SUM OF INCIDENT & REFLECTED WAVES

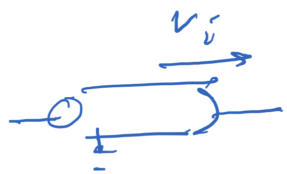
$\rightarrow V = V_i + V_R$

$\rightarrow i = i_i + i_R = \dfrac{V_i}{Z_0} - \dfrac{V_R}{Z_0}$

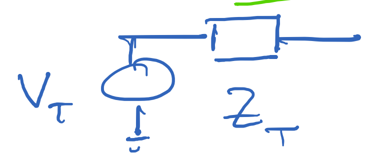
$i_i = \dfrac{V_i}{Z_0}$

$i_R = \dfrac{-V_R}{Z_0}$

WANT TO MAP



TO



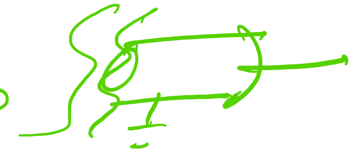
RECALL,

$V_T = V_{OC}$

$Z_T = V_{OC} / i_{SC}$

V_{oc} ?   IF LINE IS OPEN, $i = 0$

$\longrightarrow i_i = -i_R$ $i_i + i_R = 0$

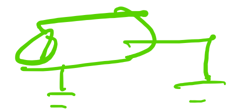


$$V_{oc} = V_i + V_R = i_i Z_0 - \underbrace{(i_R Z_0)}_{+\, i_i Z_0} = 2 i_i Z_0$$

$$= 2 V_i$$

$$= V_T$$

i_{sc} ?   @ SHORT, $V = 0$

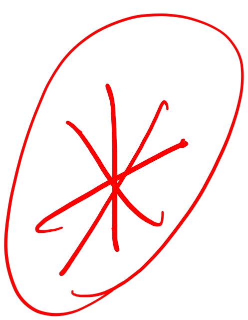


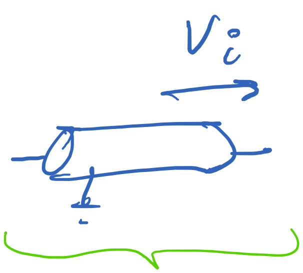
$V_i = -V_R$

$$i = \frac{V_i}{Z_0} - \frac{V_R}{Z_0} = \frac{2 V_i}{Z_0}$$

$$Z_T = \frac{V_{oc}}{i_{sc}} = \frac{2 V_i}{\left(\frac{2 V_i}{Z_0}\right)}$$

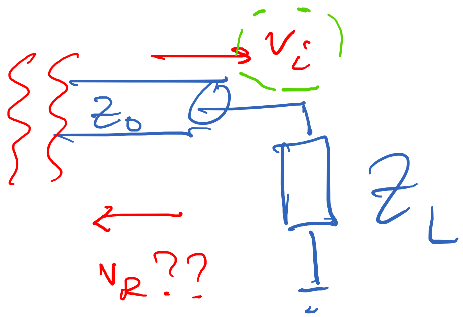
$$Z_T = Z_0$$



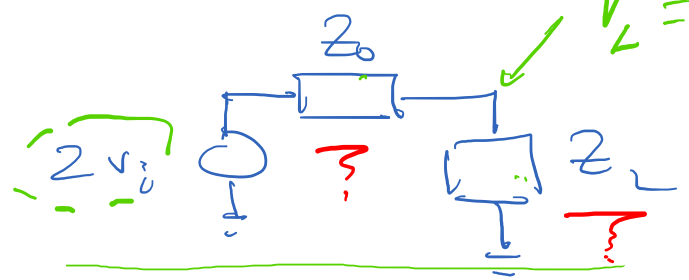


$\Longrightarrow$

REFLECTION FROM MISMATCHED LOAD



$\Rightarrow$



$V_L = V_i + V_R$

$$V_L = 2V_i \frac{Z_L}{Z_L + Z_0} = V_i + V_R \;\ldots$$

$$V_R = V_i \left[\frac{2Z_L}{Z_L + Z_0} - \frac{Z_L + Z_0}{Z_L + Z_0} \right] = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot V_i$$

$$\Gamma = \frac{V_R}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

CASE 1: $Z_L = Z_0 \longrightarrow \Gamma = 0$

CASE 2: $Z_L \longrightarrow \infty \longrightarrow \Gamma = +1$

CASE 3: $Z_L \longrightarrow 0 \quad \Gamma = -1$

IMPEDANCE LOOKING INTO MISMATCHED LINE

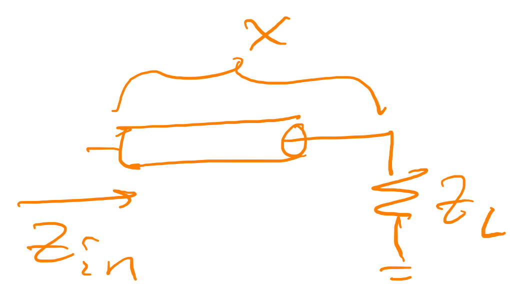
$$V = V_i \left[e^{j(kx-\omega t)} + \Gamma e^{j(-kx-\omega t)} \right]$$

$$I = \frac{V_i}{Z_0} \left[e^{j(kx-\omega t)} - \Gamma e^{j(-kx-\omega t)} \right]$$

$$Z_{in} = Z_0 \left[\frac{e^{jkx} + \Gamma e^{-jkx}}{e^{jkx} - \Gamma e^{-jkx}} \right] = Z_0 \left[\frac{1 + \Gamma e^{-2jkx}}{1 - \Gamma e^{-2jkx}} \right]$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\boxed{Z_{in} = Z_0 \, \frac{Z_L + j Z_0 \tan kx}{Z_0 + j Z_L \tan kx}}$$



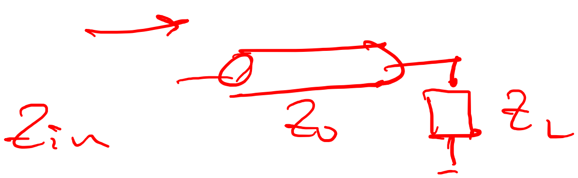
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan kx}{Z_0 + jZ_L \tan kx}$$

CASE 1 $\quad Z_L = Z_0 \implies Z_{in} = Z_0$

CASE 2 $\quad kx = \pi/2 \qquad \tan kx \to \infty$

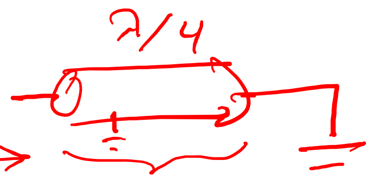
$$Z_{in} = Z_0 \times \frac{\cancel{j} Z_0 \,(\text{STH.}\ \cancel{\text{BIG}})}{\cancel{j} Z_L \,(\text{STH.}\ \cancel{\text{BIG}})} \sim \frac{Z_0^2}{Z_L}$$

$k = \frac{2\pi}{\lambda} \qquad \frac{2\pi}{\lambda} \cdot x = \pi/2 \qquad x = \lambda/4$

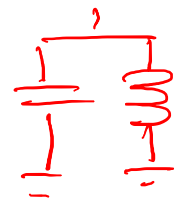


$Z_L \ll Z_0 \longrightarrow$ @ $\lambda/4 \quad Z_{in} = \frac{Z_0^2}{Z_L} \gg Z_0$

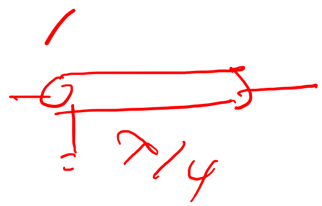
$Z_L \gg Z_0 \quad$ @ $\lambda/4 \quad Z_{in} = \frac{Z_0^2}{Z_L} \ll Z_0$



$Z_{in}(\omega_0) \rightarrow \infty$



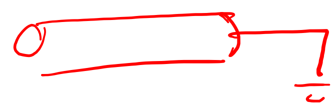
POZAR

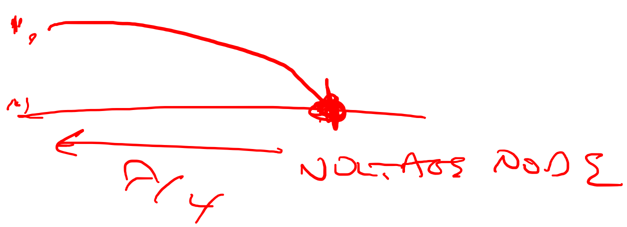


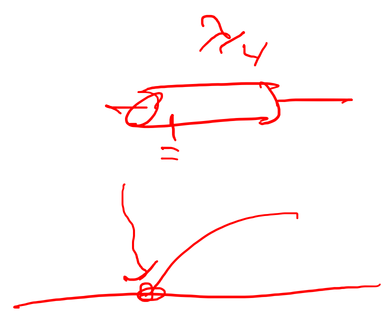
C_{eff}

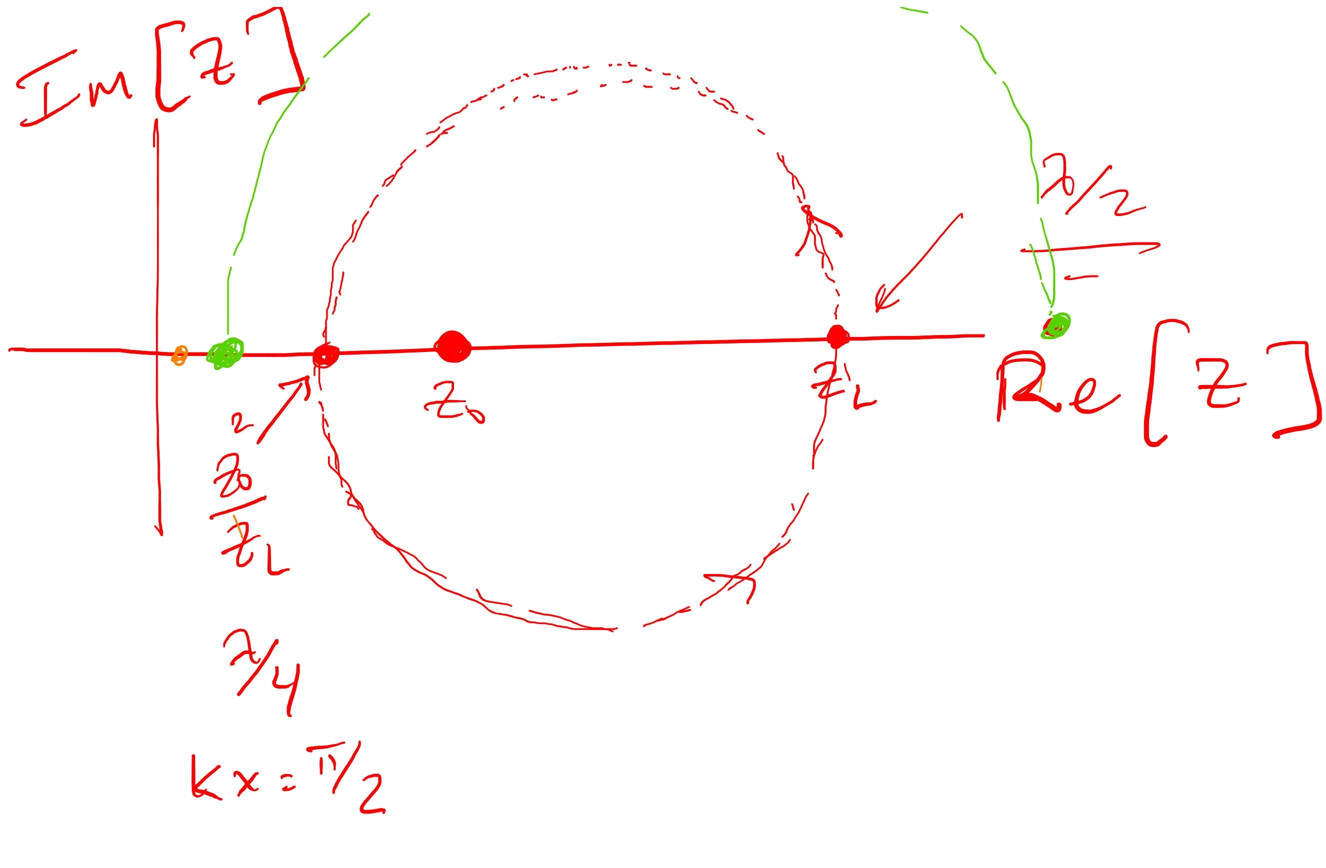
L_{eff}

$L_{eff} \approx \frac{Z_0}{\omega_0}$

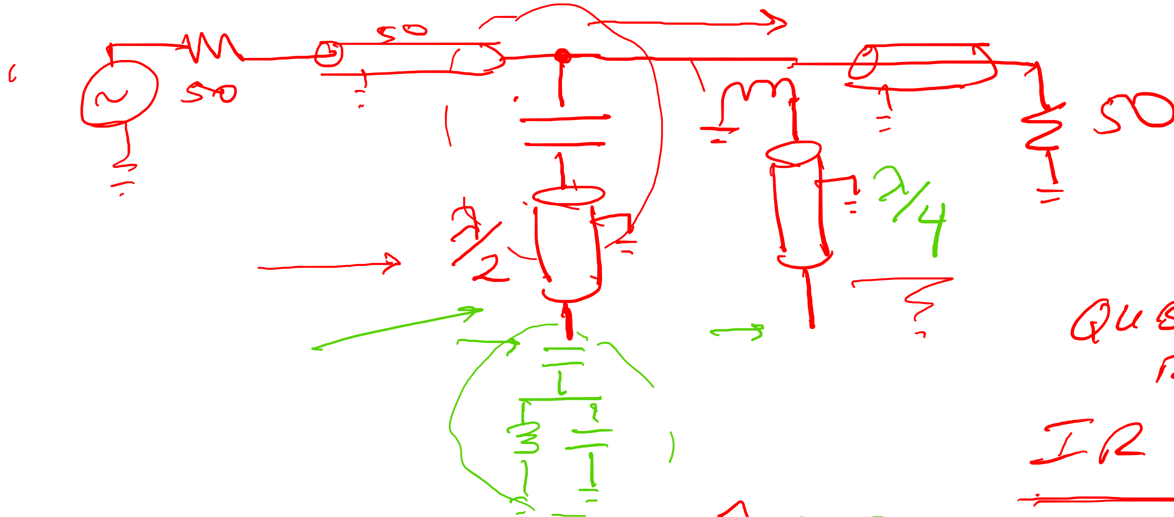








Im[z]
Re[z]
z_0
z_L
$\frac{z_0^2}{z_L}$
$\lambda/2$
$\lambda/4$
$kx = \pi/2$

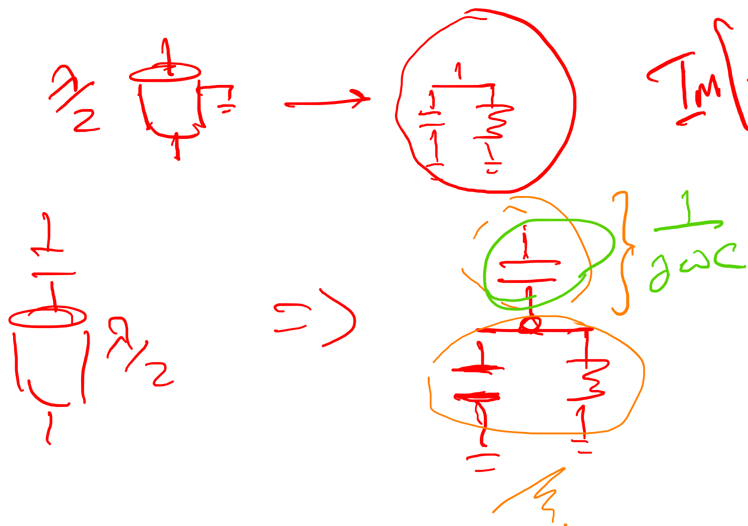


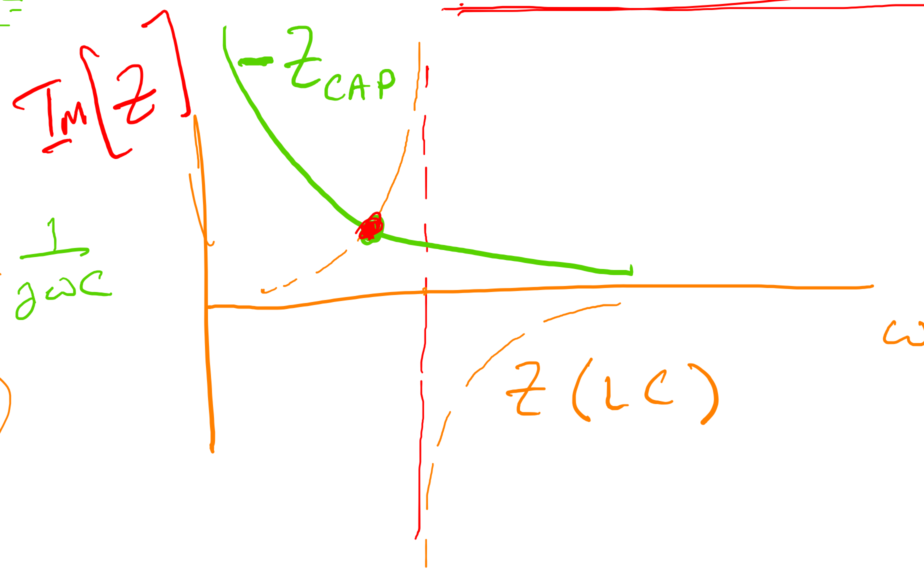
SPECTRUM ANALYZER

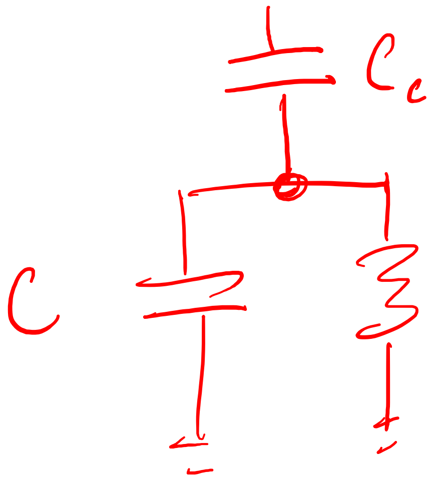
DIGITIZER

QUBIT READOUT

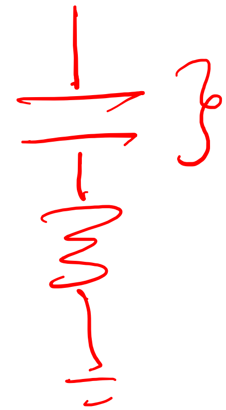
IR ASTRONOMY



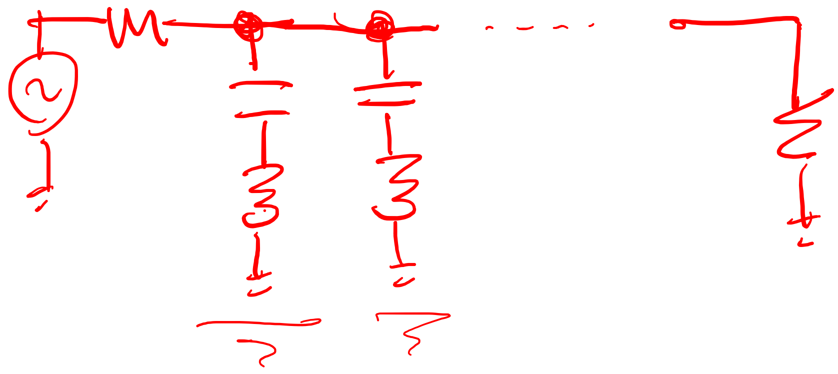




MAPS TO



$$C_{eff} = C + C_c$$



BAND-STOP FILTER