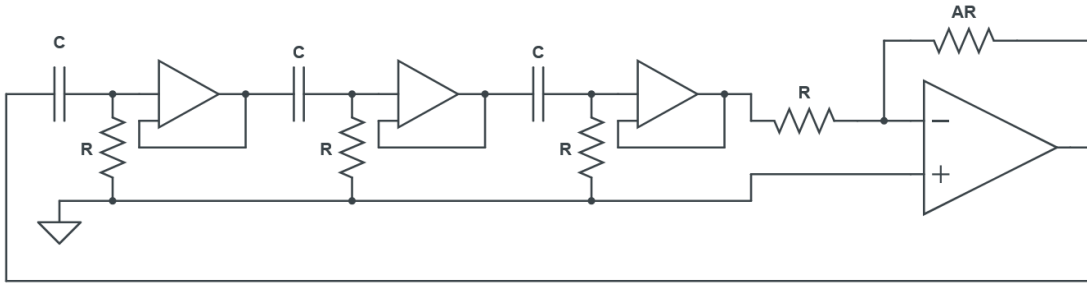


Physics 623: HW 7

PHASE SHIFT OSCILLATOR:

1. In the phase shift oscillator shown below, the three unlabeled op-amps are unity-gain buffers (gain = +1).
 - a) Find the frequency of oscillation f_{osc} in terms of $f_c = 1/(2\pi RC)$.
 - b) What is the minimum value of A such that oscillation will occur?
 - c) What is the purpose of the unity-gain buffers? In other words, how would this problem change if the buffers weren't included?



FOURIER TRANSFORMS:

2. Use the convolution theorem to prove the trigonometric identity:

$$\cos(\omega_0 t) \bullet \cos(\omega_1 t) = \frac{1}{2} (\cos((\omega_0 + \omega_1)t) + \cos((\omega_0 - \omega_1)t))$$

(This is easier to sketch and think about if you make $|\omega_0 - \omega_1|/\omega_0 \ll 1$.)

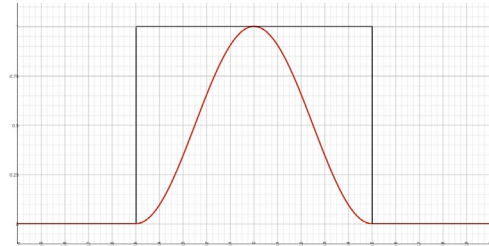
The phase detector for the lab we will do this week is effectively a multiplier that takes advantage of this to convert two frequencies into their sum and difference. The technique, called heterodyne, is also widely used in radio receivers and other instruments.

3. Use the convolution theorem to find the apparent frequency spectrum derived from a 1-second observation of the sum of two cosine waves, one at $f = 9$ Hz and one at $f = 11$ Hz. Both have amplitudes of 1 V peak. Sketch the spectrum that would be obtained. (Note that you can mathematically reproduce a one-second observation by multiplying the infinite time sequence by a rectangular function that is 1.0 between $t = -0.5$ s and $t = +0.5$ s and zero elsewhere.)

4. The sinc function that smears the spectrum observed for a finite length of time is shown in blue at the left below (compare problem 3). The oscillations can create confusing features in a spectrum that has both strong and weak sharp lines in it. The problem can be alleviated by multiplying the sampled time function by another “window” function that falls off more gently before doing the Fourier transform. One common function that is used is called a “Hanning window”. If the time sequence is observed for a time T , the Hanning window function is:

$$\frac{1}{2} \left(1 + \cos \frac{2\pi t}{T} \right) \quad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$

$$0 \quad \text{otherwise.}$$



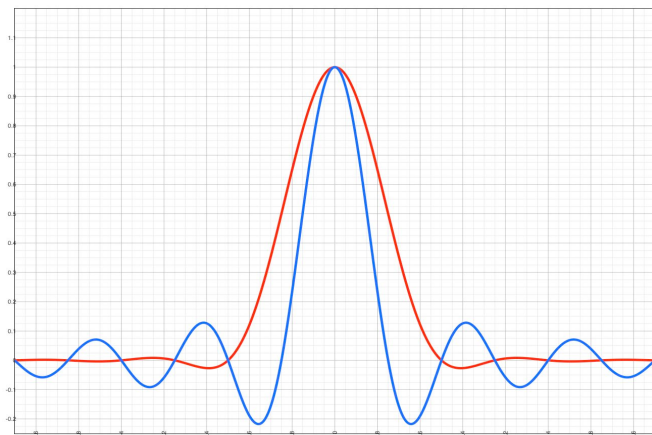
Hanning window (red) — time

This goes smoothly to zero as the ends of the observing interval are approached. The new smearing function will be the Fourier transform of this Hanning window.

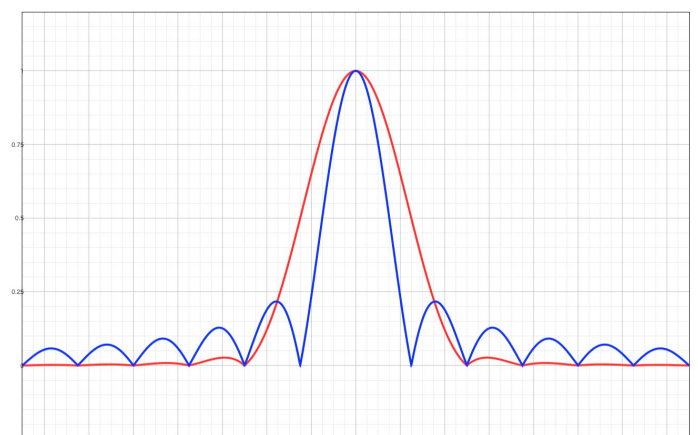
Use the convolution theorem to find the Fourier Transform of the Hanning window by constructing it as the product of a continuous $1+\cos$ multiplied by a rectangle function of length T .

Since FTs are linear, the FT of a sum is just the sum of the FTs. You need to know that the FT of a constant is a delta function at zero frequency to do the “1” part. Both terms have unit amplitude, but the cosine amplitude is split half and half between positive and negative frequencies. The result is the red curve on the left, although this won’t be obvious unless you plot your solution. You can just give the formula (if you want to check on my plot, plug in a couple of key points to check against red plot on the left below.)

Applying this type of window to a data sample is called “apodization”, or “removing the feet”. Note the tradeoff — the Hanning window greatly reduces the extraneous features far from the main response compared to the sinc function shown in blue, but there is significant loss of resolution in the main peak (it is broader). The squared values of these smearing functions are shown on the right. These are what would appear in the power spectra.



frequency



frequency