

PHYSICS 241
Spring 1993
Prof. Bernice Durand
COURSE INFORMATION

COURSE: Introduction to modern physics, lectures MWF 9:55, 3331 Sterling.

TEXT: *Elementary Modern Physics*, Paul A. Tipler, Worth Publishers 1992. There is a study guide which I didn't order at the bookstore. It looks good, but the text is already enough to read, in my opinion.

DISCUSSIONS: TUESDAYS, 1:20 or 2:25, 3331 Sterling, **MANDATORY!**

INSTRUCTOR: Prof. Bernice Durand, 4205 Chamberlin, 262-3827.

GOALS: We will master the textbook, which is a superb introduction to 20th century physics and very up-to-date. We will also experiment with different learning techniques and compare them, *which means you must attend all lectures and one regular discussion section every week.*

GRADE: There will be 42 lectures. In discussions and homework you will do work for credit averaging 2% of your grade per lecture. Thus 84% of your grade will be earned by *steady, regular mastery of the material.* A comprehensive exam on May 13 will count as 16% of your grade. Of the 84%, I estimate that 28% will be homework, much of it done in groups, 28% group work in discussions, and 28% individual work on quizzes given in discussions. I will instate hour exams only if the system doesn't seem to be working.

My usual expectations for final grades are A 90-100, AB 87-89, B 75-86, BC 72-74, C 60-71, D 50-59. I have no preconceptions as to how many of each grade I will give.

FORMAT: The 42 MWF lectures will cover the text approximately as shown on the schedule. Lectures will cover an average of 7 to 8 pages each. Some lectures will include trips to view demonstrations or apparatus, so you should be prompt.

There will be 14 Tuesday and one Friday (May 7) discussion sections, *which are essential to your learning and to your grade.* In these sessions, you will ask questions, do group problem-solving *for credit* and do individual problem-solving *for credit*, which will count as mini-exams. **DO NOT MISS DISCUSSIONS!**

Homework will be assigned on Wednesdays, in the first lecture on each chapter, and due on Wednesdays in the first lecture on the next chapter. Instructions for homework will include whether you are to work it in groups or alone. A grader will grade the homework *for credit*, with all members of a group receiving the same grade.

A comprehensive final exam *for credit* will cover the whole course.

GETTING HELP: Discussion sections and your study/working groups are for getting help. I will certainly be available after classes, MWF at 10:45 and Tues. at 3:15. I prefer to have you make an appointment with me for the next mutually agreeable time, so I don't have to sit in my office for certain hours with nobody showing up.

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PHYSICS 241, Spring 1993, Prof. Bernice Durand

COURSE SCHEDULE

WEEK	MONDAY	TUESDAY	WEDNESDAY	FRIDAY
1	No class	Jan 19 startup	Jan 20 Secs 1.1-1.3	Jan 22 Sec 1.4
2	Jan 25 Sec 1.5	Jan 26 Secs 1.1-1.5	Jan 27 Secs 1.6-1.8	Jan 29 Secs 1.9-1.10
3	Feb 1 Sec 1.11	Feb 2 Secs 1.6-1.11	Feb 3 Secs 2.1-2.2	Feb 5 Secs 2.3-2.5
4	Feb 8 Secs 2.5-2.6	Feb 9 Secs 2.1-2.6	Feb 10 Secs 3.1-3.2, App B	Feb 12 Secs 3.3-3.6
5	Feb 15 Sec 3.7	Feb 16 Secs 3.1-3.7, App B	Feb 17 Secs 3.8-3.9	Feb 19 Sec 3.10
6	Feb 22 Sec 3.11	Feb 23 Secs 3.8-3.11	Feb 24 Secs 4.1-4.2	Feb 26 Sec 4.2
7	Mar 1 Secs 4.3-4.4	Mar 2 Secs 4.1-4.4	Mar 3 Sec 4.5	Mar 5 Sec 4.6
Spring Recess	X	X	X	X
8	Mar 15 Secs 4.7-4.9	Mar 16 Secs 4.5-4.9	Mar 17 Sec 5.1	Mar 19 Secs 5.2-5.3
9	Mar 22 Sec 5.3	Mar 23 Secs 5.1-5.3	Mar 24 Secs 6.1-6.2	Mar 26 Sec 6.2, App C
10	Mar 29 Sec 6.3	Mar 30 Secs 6.1-6.3, App C	Mar 31 Secs 6.4-6.5	Apr 2 Secs 6.6-6.7
11	Apr 5 Sec 6.8	Apr 6 Secs 6.4-6.8	Apr 7 Secs 7.1-7.2	No class
12	Apr 12 Secs 7.3-7.4	Apr 13 Secs 7.1-7.4	Apr 14 Sec 7.5	Apr 16 Sec 7.6
13	Apr 19 Secs 7.5-7.6	Apr 20 Secs 7.5-7.6	Apr 21 Secs 8.1-8.3	Apr 23 Secs 8.4-8.5
14	Apr 26 Secs 8.6-8.8	Apr 27 Secs 8.1-8.8	Apr 28 Secs 9.1-9.2	Apr 30 Secs 9.3-9.5
15	May 3 Secs 9.6-9.7	May 4 Secs 9.1-9.7	May 5 Sec 9.8	May 7 Discuss Sec 9.8
16	X	X	May 12 Review Session	May 13 Thursday Exam 2:45

To: Chris Lynch
From: Srividhana Dasu

Physics 241

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PHYSICS 241 — SPRING 1994 — SYLLABUS

Professor Robert March

TEXT — *Elementary Modern Physics*, by Paul A. Tipler, Worth 1992.

Some additional readings, labeled by chapter, are included in this packet, and more may be handed out as the semester progresses.

The schedule of lectures below may be approximate, as this is the first time I have used this text in this course. Exams may cover material in lectures up to one week preceding the exam.

SCHEDULE:		
<i>Week of:</i>	<i>Lectures:</i>	<i>Discussion:</i>
Jan 24	Chapter 1 MWF	<i>get-acquainted</i>
Jan 31	Chapter 1 MWF	Problem Set 1 due
Feb 7	Chapter 2 MWF	<i>in-class problems A</i>
Feb 14	Chapter 2 M, Chapter 3 WF	Problem Set 2 due
Feb 21	Chapter 3 MWF	<i>in-class problems B</i>
Feb 28	Chapter 3 M, Chapter 4 WF	6-week exam
Mar 7	Chapter 4 MWF	<i>in-class problems C</i>
Mar 14	Chapter 4 M, Chapter 5 WF	Problem Set 3 due
Mar 21	Chapter 5 M, Chapter 6 WF	<i>in-class problems D</i>
Mar 28	SPRING BREAK	
Apr 4	Chapter 6 WF	<i>in-class problems E</i>
Apr 11	Chapter 6 MWF	Problem Set 4 due
Apr 18	Chapter 7 MWF	12-week exam
Apr 25	Chapter 7 M, Chapter 8 WF	<i>in-class problems F</i>
May 2	Chapter 8 MW, Chapter 9 F	Problem Set 5 due
May 9	Chapter 9 MWF	Essay due, Review for final

Final Exam Wednesday, May 18 at 12:25 pm

OFFICE HOURS AND EMAIL

Office hours: 10–11:45am Wed., 1:30–3pm Thu., or call for appointment.

My office is 4289 Chamberlin, the first office past the bridge from the third floor of Sterling Hall to Chamberlin. My campus phone is 262-5947. My mailbox is near the University Avenue entrance to Chamberlin.

My email (Internet) address is bobmarch@wishep.physics.wisc.edu. This is a good way to get in touch with me, and I will try to respond rapidly to questions.

DISCUSSIONS, EXAMS, AND PROBLEMS

It is important to attend discussion sections. In nearly every week, something will be going on that could affect your grade. In all weeks, the first few minutes of discussion will be devoted to questions on lectures and reading.

The 6-week and 12-week exams will be given during the discussion hour. A study guide listing the topic of each question, with suggestions for study, including problems similar to those on the exam, will be handed out a week prior to each exam. NOTE – *you may bring two sheets (written on both sides) of notes to each hour exam, four sheets to the final.*

In weeks when no problem set is due, you will divide up into groups of three or four to work problems assigned on the spot. A single copy of your solutions will be turned in, and all members of the group will receive the same grade. Membership in groups will rotate. The purpose is to teach you to study together, a skill that will serve you well in more advanced courses. In weeks when a problem set is due, we will discuss the problems. You may be called upon to work a problem at the blackboard.

GRADING

Your grade will be based 70% on exams, 30% on problem sets.

The final exam will have two parts, one cumulative and the other on the last part of the course. Each will have a separate score, making for four exam scores in all. In computing your grade, these will be *performance-weighted*: your best score will count for 30%, second-best 20%, next 12%, lowest 8%.

Individual problem sets will count for 20% of your grade, and group problems worked in class for 10%. In each case, your *worst* score will be discarded. The essay (see next page) will be lumped with problem sets for grading purposes.

There is no “curve” in this course — *i.e.* I have no preconceptions as to how many of each grade I should give. My target for grade boundaries is generally A=92, AB=89, B=80, BC=75, C=65, and D=60. I may modify these a bit, but *never* upwards — you are NOT in competition with one another!

SHORT ESSAY

At the end of this packet you will find an article by Sylvan Schweber that I believe is a perceptive analysis of the central thrust of 20th century physics, and how that thrust is likely to be modified in the 21st century. It should be read in conjunction with Chapter 8. You should write a short (~ 3-page) essay addressing the following points:

1. Did you find the article understandable and informative?
2. What (if anything) in it particularly affected your thinking?
3. Was there anything in it with which you disagreed?

Your grade on this essay will be lumped with that of the short Problem Set 1 to make a single grade. The grade will be less than perfect only if you fail to address any of the questions above.

PROBLEM SETS 1 & 2

Set 1	Due February 1	Chap 1, prob 2, 3, 12, 31, 49
Set 2	Due February 15	Chap 1, prob 33b, 39, 41, 76 Below, prob A, B Chap 2, prob 5, 7b

The problems below are based on the material on pages 4 & 5 of this packet:

Problem A — A particle has total energy 1000 MeV and momentum 870 MeV/c. Find its mass (rest energy) and its dimensionless velocity β .

Problem B — An unstable neutral particle of unknown mass decays into two particles with mass, energy, and momentum (M_1, E_1, P_1) and (M_2, E_2, P_2) . The opening angle between the particles' paths is θ . Show that the mass of the parent particle is given by

$$M = [M_1^2 + M_2^2 + 2E_1E_2 - 2P_1P_2\cos\theta]^{\frac{1}{2}}$$

LATE PAPER POLICY

Late problem sets turned in by Friday of the week assigned will get 75% credit. Anything later will get 50% credit.

SUPPLEMENTARY READINGS FOR CHAPTER 1

Dimensionless Notation (γ, β, η)

Many relativity texts employ a notation in which the coefficients in the Lorentz transformation are dimensionless ratios. The Lorentz factor $\gamma = E/mc^2$ is one of these. The other two are:

$$\beta = \frac{v}{c} \qquad \eta = P/Mc$$

The following identities are useful:

$$\eta = \beta\gamma \qquad \gamma^2 = \eta^2 + 1 \qquad \gamma = [1 - \beta^2]^{-\frac{1}{2}}$$

With this notation, the Lorentz transformation becomes:

$$x = \gamma x' + \eta ct' \qquad y = y' \qquad z = z' \qquad ct = \eta x' + \gamma ct'$$

"Natural" Units

These are sometimes referred to as "units in which $c = 1$," but this statement is not as mysterious as it seems. It simply means that in relativistic kinematics we measure both momentum and mass in energy units, i.e. the symbol " M " stands for the rest energy Mc^2 , and " P " stands for " Pc ." Since particle physics and astrophysics has no distinct units for momentum or mass, an energy unit, the electron-volt and its multiples, is employed for all three, which are related by the identity

$$E^2 = P^2 + M^2$$

In these units, momentum is stated in " eV/c ," which is equal to Pc in eV .

EXAMPLE — A π -meson has mass (rest energy) 140 MeV . If its momentum is $200 \text{ MeV}/c$, then its total energy is

$$E = \sqrt{P^2 + M^2} = \sqrt{200^2 + 140^2} \text{ MeV} = 244 \text{ MeV}$$

(A more ambitious refinement of this notation, employed in quantum field theory, has $\hbar = c = 1$, but it would only confuse matters to introduce it at this time.)

Four-dimensional Invariants

The Lorentz transformations for x and t convert what is a pure interval of time or space in one Lorentz frame is a *mixture* of a time interval and a space interval in any other Lorentz frame. In 1908 Hermann Minkowski (who had been one of Einstein's teachers in Zurich) noted that if time is treated *somewhat as if* it were a fourth dimension of space, the interval between two events is the same in all Lorentz frames. I use the phrase "somewhat as if" to emphasize that this formalism *does not* eliminate even the formal distinction between space and time, much less the psychological one: it is simply a "bookkeeping device" that gives a frame-independent representation of reality that is both appropriate and useful in relativistic systems.

Formally, it means: (1) measure both space and time in the same units, *i.e.* replace t by ct ; (2) define a modified four-dimensional pythagorean length,

$$\delta s^2 = \delta x^2 + \delta y^2 + \delta z^2 - c^2 \delta t^2$$

(that minus sign on the time term is the basis for my "somewhat as if" remark.) This interval is the same in all Lorentz frames. For example, in the case of the "lightning-struck train" in Section 1-5 of the text, subtracting the square of time interval $L_p V/c^2$ from the square of the distance between the lightning strikes in the train rest frame gives us L_p^2 , because in the platform rest frame the time interval is zero. Put another way, the length of the "four-vector" $(\delta x, \delta y, \delta z, c\delta t)$ is *invariant* under the Lorentz transformation.

What makes this formalism useful is that there are many other such four vectors, one of which is the *energy-momentum vector* (usually called the *four-momentum*, which in natural units is written

$$\vec{Q} = (P_x, P_y, P_z, E)$$

To compute the magnitude of Q , we subtract the squares of the momentum components. For a single particle, this gives the rest mass. For a system of several particles that originates from the decay of an unseen unstable particle, this invariant is the rest mass of the parent particle,

$$M^2 = (\Sigma E)^2 - |\Sigma \vec{P}|^2$$

In Problem B on Problem Set 2 you will express this quantity for the case of a two-body decay.