MC-1 Errors & Motion

MC-1a Measurement and Error

OBJECTIVES:

The major objectives of this first, short (est. 1 hr) computer lab are to begin to develop an basic understanding of what it means to make an experimental measurement and provide a methodology for assessing random and systematic errors in this measurement process. In addition this lab will also give you a minimal framework in which to introduce you to the PASCO© (page 109) interface hardware and software.

THEORY:

By now you should have had numerous opportunities to become familiar with time and the concept of a time interval. The increment of one second will be used as an intuitive reference point. In this lab you will test your ability to internalize this one second time interval by making and recording a repetitive flicking motion with your finger. By flicking your finger back and forth you will move it though an infrared beam sensor (i.e. the PASCO photogate) and each full cycle (back and forth, approximately 2 seconds) will be simultaneously recorded, plotted and tabulated by the PASCO interface software.

Your goal in this experiment is to assess the size of systematic and random errors in your data set and learn a simple methodology for distinguishing between the two.
SYSTEMATIC ERRORS: These are errors which affect the *accuracy* of a measurement. Typically they are reproducible so that they always affect the data in the same way. For instance if a clock runs slowly you will make a time measurement which is less than the actual reading.
RANDOM ERRORS: These are errors which affect the *precision* of a measurement. A process itself may have a random component (as in radioactive decay) or the measurement technique may introduce noise that causes the readings to fluctuate. If many measurements are made, a statistical analysis will reduce the uncertainty from random errors by averaging.

APPARATUS:

⇒ Computer with monitor, keyboard and mouse.

⇒ A PASCO photogate and stand: This device emits a narrow infrared beam in the gap and occluding the beam prevents it from reaching a photodetector. When the beam is interrupted the red LED should become lit. (Plugged into DIGITAL CHANNEL #1.)

⇒ A PASCO Signal Interface (CI-700 or CI-750) monitors the photodetector output *vs* time and can be configured to tabulate, plot and analyze this data.

PROCEDURE:

To configure the experiment you should refer to Fig. 1 below. Adjust the photogate so that one member can easily and repetitively flick his/her finger through the gap. The phone-jack cable from the photogate should be plugged into the DIGITAL CHANNEL #1 socket. Ignore the other sensors which may already be plugged into other sockets. It is *important* that the PASCO interface be turned on before the computer. If not the computer will not recognize it and, therefore, it must be rebooted to properly communicate with the PASCO module.

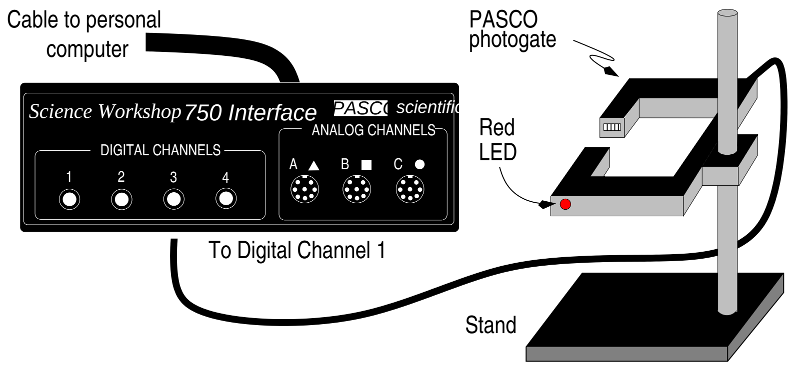

Figure 1: A schematic of the MC1a components and layout.

To initiate the PASCO (page 109) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below (web version). Fig. 2 below gives a good idea of how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do most of this experiment without necessitating any major changes.

You will note that a "dummy" first data set already exists on start-up showing a typical data run. In the table you can view all 47 data points and the statistical analysis, including mean and standard deviation. In addition there should be a plot of this data and a histogram.

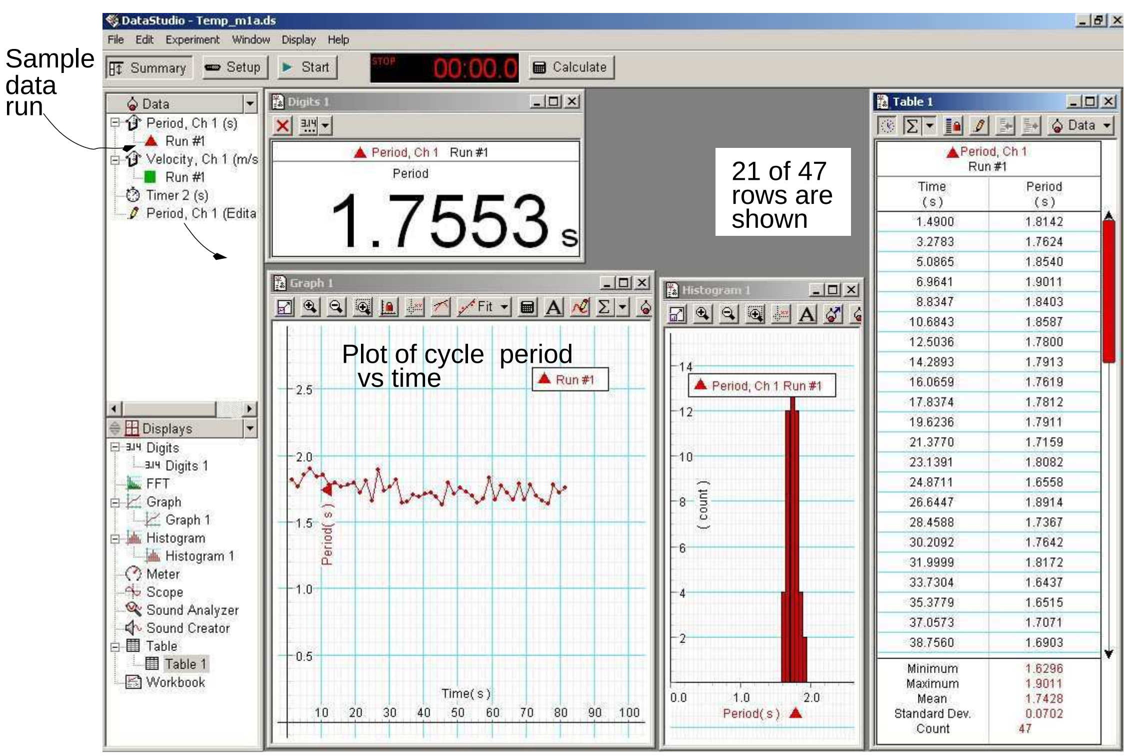

Figure 2: The PASCO Data Studio display window

SUGGESTED PROCEDURE:

1. Start the preliminaries by CLICKing on the 

icon and practice "flicking" a finger back and forth so that a two second interval appears in the window. CLICK on the **Stop** icon when done. The same person need not perform both operations. This will produce a second data set. (There is also a "monitor" function which can permit adjustments and trials without storing the results in memory. To access this type ALT-M).

2. Each run gets its own data set in the "Data" display window. (If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs.) A data set can be deleted by moving the mouse cursor to the "Run # 1" position, CLICKing the left mouse button and then striking the "Delete" key.

3. Once you are comfortable with the procedure then click on the 

icon and cycle a finger back and forth over fifty times. The click on the "Stop" icon. DO NOT watch the time display while you do this, since you want to find out how accurately and precisely you can reproduce a time interval of 2 s using only your mind.

4. What is the mean time per cycle? What is the standard deviation? The mean $\overline{t}$ and standard deviation σ are given by:

$$\overline{t} = \sum_{i=1}^{N} t_i/N \quad \text{and} \quad \sigma = \sqrt{\sum_{i=1}^{N} (t_i - \overline{t})^2/[N-1]} \quad .$$

   Questions to consider:
   I. Is your mean suggestive of a systematic error?
   II. Does your data qualitatively give the appearance of a normal distribution (i.e. a Gaussian bell curve.)

5. For analyzing and quantifying random errors, you need to asses how a data set is distributed about the mean. The standard deviation σ is one common calculation that does this. In the case of a normal distribution approximately 68% of the data points fall within $\pm 1\sigma$ of the mean (90% within $\pm 2\sigma$). Is your data consistent with this attribute?

6. Assessing the possibility of systematic behavior is somewhat more subtle. In general σ is a measure of how much a *single* measurement fluctuates from the mean. In this run you have made fifty presumed identical measurements. A better estimate of how well you have really determined the mean is to calculate the *standard deviation of the mean* $\overline{\sigma} = \sigma/\sqrt{N}$. After recording $\overline{\sigma}$ in you lab book, can you now observe any evidence that there is a systemic error in your data? Answer this same question with respect to the first "sample" data set.

7. OPTIONAL: Systematic errors can sometimes drift over time. In the best-case scenario they drift up and down so that they hopefully average out to zero. (Clearly it would be better if they could be eliminated entirely.) With respect to $\overline{t}$ and $\overline{\sigma}$ for the first 25 and second 25 cycles do you observe any systematic trends? Use the "Zoom Select" feature on the Graph. Simply by draging the mouse will highlight a subset of the data. The mean, σ and other statistical attributes will appear at the bottom of the table.

MC-1b Errors and the Density of a Solid

OBJECTIVES:

To learn about systematic and random errors; to understand significant figures; to estimate the reliability of one's measurements; and to calculate the reliability of the final result.

NOTE: This experiment illustrates the earlier sections on Errors and Significant Figures. The actual density of the metal is incidental. However, the accuracy of your estimate of reliability will show whether you have mastered the material in the earlier sections.

APPARATUS:

Metal cylinders of varying sizes, micrometer and vernier calipers, precision gauge blocks, precision balance.

Precautions:

Avoid dropping or deforming in any other way the metal cylinders. Avoid damage to the precision screw of the micrometer by turning only the friction head to open or close the caliper jaws. Be sure to disengage the caliper lock before using. (The caliper lock lets you preserve a reading.) Improper weighing procedures may damage the precision balance. Consult your instructor if in doubt. In handling the gauge blocks avoid touching the polished surfaces since body acids are corrosive.

INTRODUCTION:

First read the material on Errors and Significant Figures in the Manual Introduction). Since density is the mass per unit volume, you must measure the mass (on a balance) and compute the volume ($h\pi r^2 = h\pi d^2/4$) from measurements of the cylinder's dimensions where d is the diameter and h is the height. Any one of three length measuring devices may be used. These include a micrometer, a vernier caliper and/or a simple metric ruler. The micrometer will permit the highest precision measurements but using one can be cumbersome, especially when reading the vernier scale. All methods will demonstrate the aforementioned objectives. Your instructor will give you guidance in choosing an appropriate measurement device.

THE MICROMETER:

Record the serial number of your micrometer. Then familiarize yourself with the operation of the caliper and the reading of the scales: **work through Appendix** A. on the Micrometer. Note that use of the "friction" head in closing the jaws insures the same pressure on the measured object each time. Always estimate tenths of the smallest division. Some micrometers have verniers to assist the estimation.

THE VERNIER CALIPER:

Work through Appendix A. on the Vernier Caliper. You may also wish to try this java applet vernier at http://webphysics.davidson.edu/. Experiment with one of the large verniers in the lab until you are sure you understand it. Note that verniers need not be decimal: for many inch scales the vernier estimates 1/8's of the 1/16 inch division, i.e. 1/128's of an inch. However vernier calipers divide the inch into 50 divisions and the vernier estimates 1/25 of the 1/50 inch divisions, i.e. 1/1000 inch or 1 mil. The vernier was invented 1631 by Pierre Vernier.

Precautions on use of the calipers:

1. Unclamp **both** top thumbscrews to permit moving caliper jaws.
2. Open caliper to within a few mm of the dimension being measured.
3. Close right thumbscrew to lock position of lower horizontal knurled cylinder which executes fine motions of caliper jaw. **Never over tighten!**

CALIBRATION OF THE MICROMETER (or VERNIER, ETC.):

1. OPTIONAL: Wipe the micrometer caliper jaws with cleaning paper. Then determine the zero error by closing the jaws. Make and record five readings. The variation of these repeated readings gives you an estimate of the reproducibility of the measurements. (For those using the micrometers they have been given a small zero error. Thus a zero error correction is necessary.) In general any measurement device can have a zero error.
2. Measure all four calibration gauge blocks (6, 12, 18 and 24 mm): Set the gauge blocks on end, well-in from the edge of the table, and thus freeing both hands to handle the caliper. Record the actual (uncorrected) reading. A single measurement of each block will suffice.
3. Plot a correction curve for your micrometer, i.e. plot errors as ordinates and nominal blocks sizes (0, 6, 12, 18, and 24 mm) as abscissa. Normally the correction will not vary from block to block by more than 0.003 mm (for the micrometer). If it is larger, consult your instructor.

DENSITY DETERMINATION:

1. Make five measurements (should be in millimeters) of the height and five of the diameter. Since our object is to determine the volume of the cylinder, distribute your measurements so as to get an appropriate average length and average diameter. Avoid any small projections which would result in a misleading measurement. If not possible to avoid, estimate their importance to the result. Record actual readings and indicate, in your lab book, how you distributed them.
2. Calculate the average length, average diameters and the respective standard deviation.
3. Use your correction curve to correct these average readings. If you were to use the uncorrected values, how much relative error would this introduce?
4. Weigh the cylinder twice on the electronic balance; estimate to 0.1 mg.
5. From the average dimensions and the mass, calculate the volume and density. Make a quantitative (refer to the Error and Uncertainties section on page 12) estimate of the uncertainty in the density. The sample worksheet asks for both the maximum and minimum values. You should use, as your starting point:

$$\text{Density}(\rho \pm \Delta\rho) = \rho(h \pm \Delta h, d \pm \Delta d, m \pm \Delta m) = \frac{m}{\pi(d/2)^2\ h}$$

6. Compare the density with the tabulated value. Tabulated values are averages over samples whose densities vary slightly depending upon how the material was cast and worked; also on impurity concentrations.

7. In your notebook or lab form summarize the data and results. Also record your result on the blackboard. Is the distribution of blackboard values reasonable, i.e. "normal" distribution (refer to the section on page 10)?

   To test how accurately you can estimate a fraction of a division, estimate the fractions on the vernier caliper before reading the vernier. Record both your estimate and the vernier reading.

Related facts and URL links:

Question: Why are there are *exactly* 25.4 mm in 1 inch?

Answer(not verified): The Treaty of the Meter (Convention du Metre) in the late 19th century established the first centralized international system of metrology. This defined the meter.

In 1959, the countries of the world that were using Imperial units defined them uniformly based on the metric units. The inch was simply defined that way and agreed to by all. Before 1959, different countries related inches to meters in other ways. Among them was the United States. The Metric Act of 1866 defined the meter in terms of inches (i.e., before the Treaty of the Meter), and that relationship had continued to been used even after the Treaty fixed the length of the meter.

Changing the definition in the U.S. in 1959 caused very little problem, except for the U.S. Geological Survey. When you deal with things 10^5 meters big (like the sizes of the states), even 1 part in 10^5 changes affect the specifications of boundary lines in significant ways. So, to this day the U.S. has two different systems of inches, feet, and yards (in the ratio of 36:3:1, for both). There's the usual inch, foot, and yard; and there's the survey foot (and inch and yard), which is based on the pre-1959 definition. (The ambiguity goes away, of course, when metric specification is used. Newer USGS maps are metric.)

Links to metrology site(s):
National Institute of Science & Technology at http://www.nist.gov/

Experiment 1b Worksheet

1.

CALIBRATION TABLE			
Micrometer Seriel # =	______		
MEASUREMENTS			
Zero gap 1 =	______	±	______
2 =	______		
3 =	______		
4 =	______		
5 =	______		
Mean$\pm\sigma$=	______	±	______
Gauge block 6 mm	______		
12 mm	______		
18 mm	______		
24 mm	______		

2. Make a plot of Gauge block thickness (x-axis) versus Micrometer reading.
   Next fit the data to a line; this generates a correction curve where
   Corrected value = actual × slope + intercept
   slope = ______
   intercept = ______

3. Now for the unknown value

CYLINDER	Height (±______)	Width (±______)	Mass (±______)
1=	______	______	______
2=	______	______	______
3=	______	______	______
4=	______	______	______
5=	______	______	______
Mean$\pm\sigma$ =	______ ± ______	______ ± ______	______ ± ______
Corrected Mean$\pm\sigma$=	______ ± ______	______ ± ______	______ ± ______
Density =	______		
Max. Density =	______		
Min. Density =	______		

4. Final result, density = ______ ± ______

5. Cut and tape this into you lab notebook.

6. Answer the following questions in your notebook.
   A. Identify two sources of systematic error and give their magnitude.
   B. Identify two examples of random error and give their magnitude.

MC-1c Motion, Velocity and Acceleration

OBJECTIVES:

The major objectives of this exploratory computer lab are two-fold. Since you will be using computer based data-acquisition throughout this course, we expect you to become familiar with the PASCO© (page 109) interface hardware and software. Our second objective is for you to develop an intuition for Newtonian mechanics by experimenting with 1-D motion. With the exception of the first part, calibration of the sonic sensor, there is no extensive write-up in this lab, but only a series of recommended experiments and the requirement to write down your observations in your lab book/form.

THEORY:

The motion of an object is described by indicating its distances x_1 and x_2 from a fixed reference point at two different times t_1 and t_2. From the change in position between these two times one calculates the average *velocity* (remember that direction is implied) for the time interval:

$$\text{average velocity} \equiv \overline{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}\ m/s$$

The *acceleration* of an object is found by finding its velocity v_1 and v_2 at two different times t_1 and t_2. From the change in velocity between two different times one calculates the average acceleration for the time interval:

$$\text{average acceleration} \equiv \overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}\ m/s^2$$

FUNDAMENTAL CONCEPTS:

1. The equation that describes the motion of an object that moves with *constant velocity* is: $x = A + B \cdot t$.
   If you make a plot of x versus t, you find that it describes a straight line. The letter A indicates the position of the object at time $t = 0$. The letter B is the slope of the line, and is equal to the velocity of the object. So we can rewrite this $x = x_0 + v \cdot t$.

2. The equation that describes the motion of an object that moves with *constant acceleration* is: $x = x_0 + v_0 \cdot t + \frac{1}{2}at^2$ and $v = v_0 + at$.
   So we can rewrite this as: $x = A + Bt + Ct^2$ and $v = B + 2Ct$. The letter A indicates the position of the object at time $t = 0$. The letter B is the the velocity of the object at time $t = 0$, and is the slope of the graph at this time. The letter C is equal to half the acceleration.

PRECAUTIONS:

In order for the position sensor to work properly it must be pointed in such away that it "sees" the vane, and doesn't identify the front of the cart; that means that it must be pointed slightly upwards. The sonic ranger tends to "see" the closest reflecting surface. In addition the minimum range is approximately 40 *cm*.
Make sure you do not drop the carts or allow them to roll off the table, because it

damages the bearings and they begin to suffer too much friction. Try to arrange to keep the cart on the track all the time.

APPARATUS

⇒ Computer with monitor, keyboard and mouse.

⇒ A PASCO position sensor; this device emits a series of short pulses of sound, and receives the echo of the sound reflected by a nearby object. The length of the time interval between the emission and the reception of the sound pulse depends on the distance to the reflecting object.
This method of locating an object is the same as the one used by bats to find flying insects or by navy ships to locate submarines.

⇒ A PASCO Signal Interface converts the time interval between the emission and reception of the sound pulse to digital form, i.e., numbers that can be then plotted on the monitor.

⇒ PASCO dynamic track with magnetic bumpers; cart with reflecting vane; meter stick; one or two steel blocks.

PROCEDURE:

Your instructor will demonstrate how to configure the experiment. To initiate the PASCO(page 109) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below. The Fig. 1 below shows how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do this experiment without necessitating any changes. All three measured quantities, position, velocity, and acceleration, are displayed simultaneously. Since velocity is determined from the position data and acceleration from the velocity the "scatter" in the data will become progressively more pronounced.

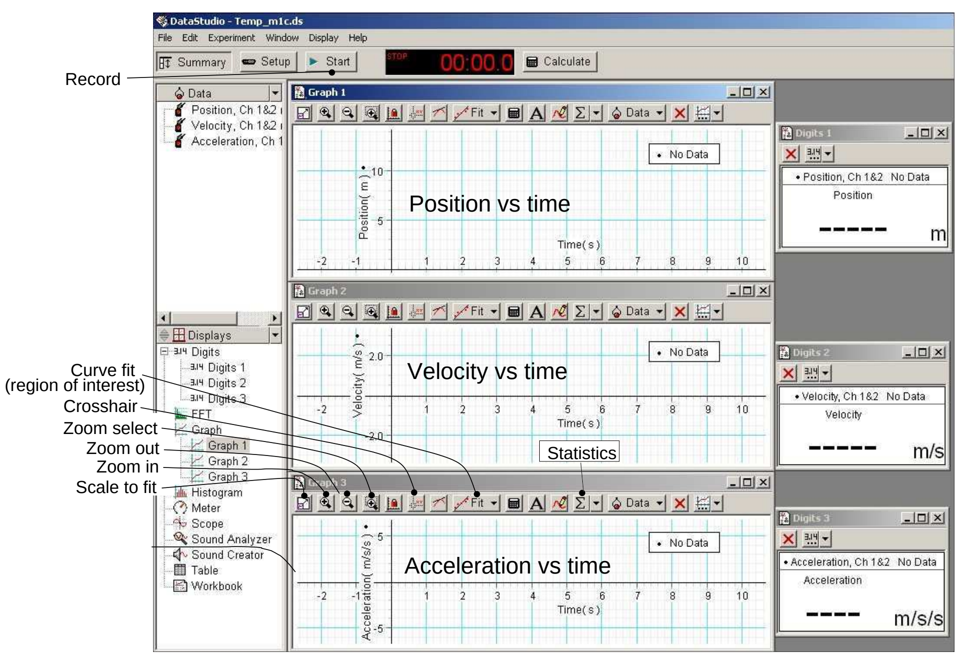

Figure 1: The PASCO Data Studio display format

Experiment I, Basic Operation and Sonic ranger calibration:

1. To start the data acquisition CLICK on the START icon. To stop it CLICK on the same stop which will now become the STOP icon. Each run gets its own data set in the "Data" display window. If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs, unless you clear (delete) all data. The is also a monitor feature that can be accessed on the EXPERIMENT pull down menu (or Alt-M).
2. With the data acquisition started move the cart *to* and *fro* and watch the position, velocity and acceleration displays.
3. STOP the data acquisition.
4. CLICK once (or twice) on the SCALE TO FIT icon. CLICK on the ZOOM SELECT icon and move to one of graph regions and CLICK and DRAG the mouse. CLICK on the CROSSHAIR icon and move about on the various graphs. Change the plot display region be manual adjusting the x-min, x-max, etc. values. To do this double CLICK within a plot window. CLICK on the STATISTICS icon once and then again. *Make* sure that all members of the group have an opportunity to test these components. It will facilitate the rest of the lab course if the basic operations on the software interface are understood by everyone!
5. DELETE the data set by a CLICK on the RUN #1 item in the "Data" window and then striking the "Delete" (<DEL>) key.
6. Start the data acquisition and observe the closest distance to the sonic ranger at which it still functions. This value is supposed to be close to 40 cm. If it is much larger, re-aim the position sensor.
7. Configure the distances so that when the cart nearly touches the near magnetic bumper the sonic ranger still records accurately.
8. Measure the position at two distances approximately 1 m apart and compare the printed centimeter scale with the position sensor readout. By how much do the readings differ?

Experiment II, Inclined Plane and Motion:

1. Raise the side of the track furthest from the position sensor using one of the supplied blocks. Hang the 500 g weight off the other side to weigh the front end down and prevent unwanted slippage of the track (See the demonstration set-up to see how the weight should be positioned.)
2. Find an appropriate release point that allows the cart to roll down the track *without* striking the magnetic bumper.
3. CLICK the START button and release the cart letting it bounce three or four times and the CLICK the STOP button.
4. Qualitatively describe the shape of the three curves: position, velocity and acceleration and discuss how they evolve with time.

5. Obtain a hard copy of this data by simultaneously pressing the **CTRL-p** keys.

6. Label/identify the various key features in the various curves by writing directly on the hard copies. Paste the graphs in your note book.

QUESTIONS: (to be discussed as a group)

1. Does the velocity increase or decrease linearly with time when it is sliding up or down the track?

2. When the velocity is close to zero can you observe any discrepancies in the data? Can you think of a reason for any deviations from linearity?

3. Why does the maximum of the position readout fall with each subsequent bounce?

4. Is the collision with the magnetic bumper or the residual friction in the bearing the most likely source of loss?

5. Can you think of a method using your data to determine which of these two proposed mechanisms is the most likely culprit?

Experiment III, Reproducing Expt. II manually:

1. Remove the block so that the cart is no longer raised.

2. Start a new data set (by clicking the START button) and try to move the cart back and forth at constant velocity using your hand from the side. Alternatively have one team member aim the position sensor at another member holding a book waist-high and walk towards the sensor or away from the sensor.

3. Using the cross hair estimate your velocity in either m/s or cm/s.

4. Repeat step 3. but now try to obtain a region of constant acceleration.

Experiment IV, Acceleration at g (9.8 m/s^2):

1. Have one member of the group stand *carefully* on a chair and hold the position sensor facing downward above another member's head while he or she is holding a notebook on their head.

2. Start a new data set and have the student holding the notebook jump up and down a few times.

3. Stop the recording and determine whether free-fall yields a constant acceleration close to the accepted value.

4. You may wish to try to fit data and measure the acceleration. First use the ZOOM SELECT icon to choose a range of data. The CLICK on the FIT icon to get at the pull down menu. You could use the position data and use a quadratic function or the velocity data and use a linear fit (or even the acceleration curve). Ask you lab instructor for a demonstration if you are unsure.

5. Fini.