

HW 12

1) BD 11.2

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2} P + \frac{1}{2} m \omega^2 r^2 P = E P$$

$$p = r \sqrt{\frac{m \omega}{\hbar}} \quad E = \hbar \omega \epsilon$$

$$-\frac{\hbar^2}{2m} \frac{m \omega}{\hbar} \frac{d^2 P}{dp^2} + \frac{m \omega}{\hbar} \frac{\hbar^2 l(l+1)}{2m p^2} P + \frac{1}{2} m \omega^2 \frac{\hbar}{m \omega} p^2 = \hbar \omega \epsilon P$$

$$-\frac{d^2 P}{dp^2} + \frac{l(l+1)}{p^2} P + p^2 P - 2\epsilon P = 0$$

b) $\epsilon = 2n' + l + 3/2$

let $n = 2n' + l \Rightarrow \epsilon = n + 3/2$

$n' = 0 \dots$

$l=0$	$n=0, 2, \dots$	$0s, 2s, 4s, \dots$
$l=1$	$n=1, 3, \dots$	$1p, 3p, 5p, \dots$
$l=2$	$n=2, 4, \dots$	$2d, 4d, 6d$

c) $|001\rangle, |100\rangle, |010\rangle$ are the three $1p$ $m = -1, 0, 1$ levels (span the same space)

2) 11.4 If $[H, L_z] = 0 \Rightarrow [H, L_{\pm}] = 0$

$\therefore L_{\pm}|m\rangle$ is eigenstate w/ same energy as m but $L_{\pm}|m\rangle = |m\pm 1\rangle$. Same for L_{\mp} , so

all states have energies independent of m

b) not true; explicitly violated by particle in a sphere

11.6 a, 1st part of b see Mathematica

b cont

$$A_l^+ H_l u_l = A_l^+ (A_l^- A_l^+ u_l + \frac{1}{(l+1)^2} u_l)$$

$$\in A_{l+1}^+ u_l = (H_{l+1} - \frac{1}{(l+1)^2}) A_l^+ u_l + \frac{1}{(l+1)^2} A_l^+ u_l$$

$$= H_{l+1} A_l^+ u_l \quad \checkmark$$

$$c) A_{l-1}^+ A_{l-1}^- = A_l^- A_l^+ + \frac{1}{(l+1)^2} - \frac{1}{l^2}$$

$$A_{l-1}^- H_l u_l = A_{l-1}^- (A_{l-1}^+ A_{l-1}^- + \frac{1}{l^2}) u_l$$

$$\in A_{l-1}^- u_l = (H_{l-1} + \frac{1}{l^2}) A_{l-1}^- u_l + \frac{1}{l^2} A_{l-1}^- u_l$$

$$= H_{l-1} A_{l-1}^- u_l$$

$$d) \langle u_l | A_l^- A_l^+ | u_l \rangle = \int d\rho u_l^* A_l^- A_l^+ u_l = (\epsilon - \frac{1}{(l+1)^2})$$

$$\langle u_l | A_l^- = (A_l^+ u_l)^* = -(A_l^+ u_l)^*$$

$$\infty \int d\rho u_l^* A_l^- A_l^+ u_l = \int d\rho |A_l^+ u_l|^2 < 0$$

$$\infty \quad \epsilon - \frac{1}{(l+1)^2} < 0 \Rightarrow \epsilon < \frac{1}{(l+1)^2}$$

e) can increase l by repeatedly applying A_l^+ .

At some l_{\max} , this must terminate. This must happen at $\frac{1}{(l+1)^2} \rightarrow \infty \quad \epsilon = \frac{1}{(l_{\max}+1)^2} = \frac{1}{n^2}$

$$A_{l_{\max}}^+ u_{l_{\max}} = 0 \Rightarrow \left(\frac{1}{d\rho} - \frac{n}{\rho} + \frac{1}{n} \right) u_{l_{\max}} = 0$$

11.6f) soln to eqn

$$\text{is } u_{lmax} = \rho^n e^{-\rho/h}$$

repeatedly apply A_{l-1}^- to generate the wavefunctions

$$11.7) -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2} \psi + \left(\frac{A}{r^2} - \frac{B}{r} \right) \psi = E \psi$$

$$\text{let } \frac{\hbar^2 l(l+1)}{2m} = \frac{\hbar^2 l'(l'+1)}{2m} + A \quad B \leftrightarrow e^2$$

$$E = -\frac{1}{2} \left(\frac{B}{\hbar c} \right)^2 \frac{mc^2}{(n_r + l' + 1)^2}$$

$$= -\frac{B^2 m}{\hbar^2 (n_r + l' + 1)^2}$$

$$l' = -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{8mA}{\hbar^2} + 1 + 4l(l+1)}$$

5) Mathematica

$$6) \text{ large } r \quad P \sim e^{-Kr} = e^{-\sqrt{\frac{-2mE}{\hbar^2}} r}$$
$$= e^{-P/n^*}$$

where $E = -\frac{R_H}{n^{*2}}$

∴ effective just plot w/ ~~real~~ Bohr radius

$$P \rightarrow n \frac{P}{n^*}$$

$$n^* = \sqrt{\frac{13.6 \text{ eV}}{4.18 \text{ eV}}} = 1.8$$

$$\text{In[64]}:= \mathbf{H_{1_}[f_]} := \partial_{\rho,\rho} f + \left(-\frac{1(1+1)}{\rho^2} + \frac{2}{\rho} \right) f;$$

$$\mathbf{Am_{1_}[f_]} := \partial_{\rho} f + \left(\frac{(1+1)}{\rho} - \frac{1}{1+1} \right) f;$$

$$\mathbf{Ap_{1_}[f_]} := \partial_{\rho} f - \left(\frac{(1+1)}{\rho} - \frac{1}{1+1} \right) f;$$

11.6 a

$$\text{In[67]}:= \mathbf{H_1@u[\rho]} == \mathbf{Am_1@Ap_1@u[\rho]} + \frac{1}{(1+1)^2} u[\rho] // \mathbf{Simplify}$$

Out[67]= True

b

$$\text{In[68]}:= \mathbf{Ap_1@Am_1@u[\rho]} == \mathbf{Am_{1+1}@Ap_{1+1}@u[\rho]} + \left(\frac{1}{(1+2)^2} - \frac{1}{(1+1)^2} \right) u[\rho] // \mathbf{Simplify}$$

Out[68]= True

$$\text{In[71]}:= \mathbf{u[\rho] Am_1@Ap_1@u[\rho]} // \mathbf{Simplify}$$

$$\text{Out[71]}= -\frac{u[\rho] \left((1+31^3+1^4+1^2(3-2\rho)-41\rho+(-2+\rho)\rho) u[\rho] - (1+1)^2 \rho^2 u''[\rho] \right)}{(1+1)^2 \rho^2}$$

$$\text{In[72]}:= \mathbf{Solve[1p(1p+1) == 1(1+1) + \frac{2mA}{\hbar^2}, 1p]}$$

$$\text{Out[72]}= \left\{ \left\{ 1p \rightarrow \frac{-\hbar^2 + \sqrt{8Am\hbar^2 + \hbar^4 + 41\hbar^4 + 41^2\hbar^4}}{2\hbar^2} \right\}, \left\{ 1p \rightarrow -\frac{\hbar^2 + \sqrt{8Am\hbar^2 + \hbar^4 + 41\hbar^4 + 41^2\hbar^4}}{2\hbar^2} \right\} \right\}$$

Prob 5

$$\text{In[76]}:= \mathbf{Am_0@Am_1@Am_2@Am_3@(\rho^5 \text{Exp}[-\rho/5])} // \mathbf{Simplify}$$

$$\text{Out[76]}= \frac{63}{625} e^{-\rho/5} \rho (9375 - 7500\rho + 1500\rho^2 - 100\rho^3 + 2\rho^4)$$

$$\text{In[77]}:= \mathbf{H_0@ \%} == \frac{1}{25} \% // \mathbf{Simplify}$$

Out[77]= True

Prob 6

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In[100]:= Plot[ $\frac{63}{625} e^{-\rho/5} \rho (9375 - 7500 \rho + 1500 \rho^2 - 100 \rho^3 + 2 \rho^4)$  /.  $\rho \rightarrow \rho p^5 / 1.8$ , { $\rho p$ , 0, 55}]
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