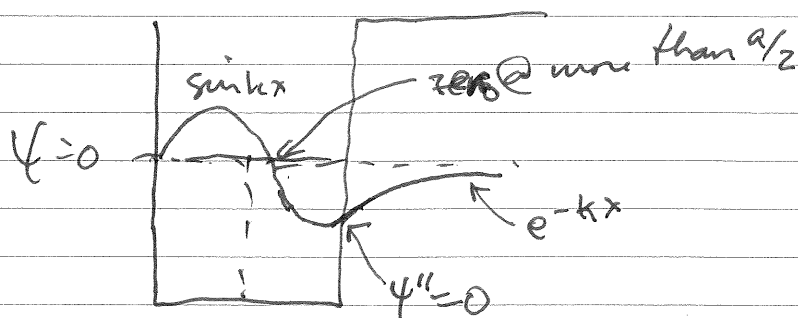
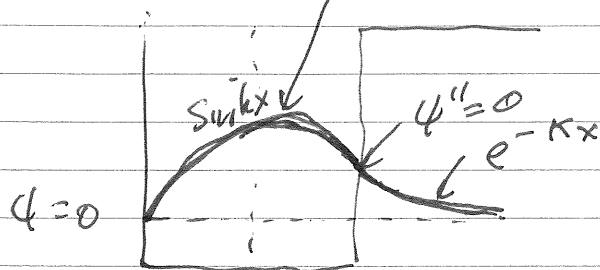


Exam 1 Solus

1a) grnd state



b) to smoothly match sin to exp, need more than $(n - \frac{1}{2})\pi$ of phase for the n th state

$$(n - \frac{1}{2})\pi = ka = \sqrt{\frac{2mV_0}{\hbar^2}} a = \phi$$

$$\therefore n = \text{Floor}\left(\frac{\phi}{\pi} + \frac{1}{2}\right)$$

c) $\psi_I = \sin kx$ $\psi_{II} = \sin ka e^{-K(x-a)}$

$$k \cos ka = -K \sin ka$$

$$k \cot ka = -K$$

plot $k \cot ka$ vs E , $k = \sqrt{\frac{2m(E)}{\hbar^2}}$
plot $-K$ vs E , $-K = -\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$

find intersection

$$2) \quad -\frac{\hbar^2}{2m} \psi'' + \frac{\hbar^2 g}{2m} \delta(x) \psi = \frac{\hbar^2 k^2}{2m} \psi$$

$$\text{left } \psi_L = e^{ikx} + r e^{-ikx}$$

$$\text{right } \psi_R = t e^{ikx}$$

$$\psi_L(0) = \psi_R(0) \Rightarrow 1 + r = t$$

$$-(\psi'_R(0) - \psi'_L(0)) + \underset{\substack{\uparrow \\ \psi(0)}}{g} t = 0$$

$$-ikt + ik(1-r) + g t = 0$$

$$1 - (t - 1) = 2 - t$$

$$-ikt + 2ik - ikt + g t = 0$$

$$t = \frac{2ik}{2ik - g}$$

$$t = \frac{4k^2}{4k^2 + g^2}$$

$$3) \quad \langle x^2 \rangle = \frac{2 \int_0^\infty x^2 e^{-2Kx} dx}{2 \int_0^\infty e^{-2Kx} dx} = \frac{1}{4K^2} \frac{\int_0^\infty u^2 e^{-u} du}{\int_0^\infty e^{-u} du} = \frac{1}{2K^2}$$

$$\langle p^2 \rangle = 2m \langle E \rangle = -\hbar^2 K^2 = 2m \left(-\frac{\hbar^2 K^2}{m} \right) = \hbar^2 K^2$$

$$\Delta p \Delta x = \sqrt{\langle p^2 \rangle \langle x^2 \rangle} = \sqrt{\frac{\hbar^2}{2}} = \frac{\hbar}{\sqrt{2}}$$

$$-\frac{\hbar^2}{2m} \psi'' + V_0 \left(\frac{x}{a}\right)^4 \psi = E \psi$$

$$-\frac{1}{2} \psi'' + \frac{mb^2}{\hbar^2} \left(\frac{x}{a}\right)^4 V_0 \psi = \frac{mb^2 E}{\hbar^2} \psi$$

$$\frac{mb^6}{\hbar^2 a^4} V_0 = 1$$

$$b = \left(\frac{\hbar^2 a^4}{m V_0} \right)^{1/6}$$

energy scale

$$\frac{\hbar^2}{mb^2} = \frac{\hbar^2}{m \left(\frac{\hbar^2 a^4}{m V_0} \right)^{1/3}} = \left(\frac{\hbar^2}{m} \right)^{2/3} \frac{V_0^{1/3}}{a^{4/3}}$$

$$4) \int_{-x_T}^{x_T} k dx = \pi$$

$$V_0 \left(\frac{x_T}{a} \right)^4 = E \Rightarrow x_T = a \left(\frac{E}{V_0} \right)^{1/4}$$

$$k = \sqrt{\frac{2m(E-V)}{\hbar^2}} = \sqrt{\frac{2mE}{\hbar^2}} \sqrt{1 - V/E} = \sqrt{\frac{2mE}{\hbar^2}} \sqrt{1 - \frac{V_0}{E} \frac{x^4}{a^4}}$$

$$= \sqrt{\frac{2mE}{\hbar^2}} \sqrt{1 - \left(\frac{x}{x_T} \right)^4}$$

$$\int_{-x_T}^{x_T} k dx = \sqrt{\frac{2mE}{\hbar^2}} x_T \int_{-x_T}^{x_T} \sqrt{1 - \left(\frac{x}{x_T} \right)^4} \frac{dx}{x_T}$$

$$= \sqrt{\frac{2mE}{\hbar^2}} a \left(\frac{E}{V_0} \right)^{1/4} (\sim 1.5) = \pi$$

$$E \approx V_0 \left[\frac{\pi}{1.5} \left(\frac{\hbar^2}{2m} \right)^{1/2} \frac{1}{a} \right]^4$$

$$= V_0 2^4 \cdot \left(\frac{\hbar^2}{2ma^2} \right)^2$$

$$= 16 V_0 \left(\frac{(1973 \text{ eV}\cdot\text{\AA})^2}{2(87)(9315 \times 10^6 \text{ eV})(10^{-10} \text{ \AA})^2} \right)^2$$

$$E \approx V_0 \left[\frac{\pi}{1.5} \left(\frac{\hbar^2}{2mV_0 a^2} \right)^{1/2} \right]^4 = V_0 \cdot (16) \left(\frac{\hbar^2}{2mV_0 a^2} \right)^2$$

$$= 16 \frac{\hbar^4}{2m a^4} = 0.5 \mu\text{K}$$

$$\text{or, } \langle x^2 \rangle \approx \frac{x_T^2}{2} \quad \langle p^2 \rangle = \frac{2mE}{2} = mE$$

$$\frac{x_T^2}{2} mE \sim \hbar^2 \quad \frac{2}{2} \left(\frac{E}{V_0} \right)^{1/2} mE = \hbar^2$$

$$E \approx \left(\frac{\hbar^2}{mV_0 a^2} \right)^{2/3} V_0$$

$$\left(\frac{2\hbar^3}{ma^2V_0} \right)^{2/3} V_0 = E = 5 \mu K$$