Phys 448 HW 2

1) Consider a beam of particles with known kinetic energy *E*. Assume that the wavefunctions for these particles can be written $\Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar}$ and, starting with the Schrodinger equation for free particles, find a new wave equation for $\psi(\mathbf{r})$ that involves only spatial derivatives.
2) The beam is travelling in the z-direction. Assume that the spatial wavefunction can be written $\psi(\mathbf{r})=f(x,y,z)e^{ikz}$, where $\partial_z f \ll kf$. Plug this into your equation from Prob. 1, and use the deBroglie relation between *k* and *E* to find a new "paraxial" wave equation for *f*. It should involve a single derivative with respect to z, and $\nabla_\perp^2 f \equiv \partial_x^2 f + \partial_y^2 f$.

3) Use Mathematica to verify that the function $f = \dfrac{1}{q(z)} e^{ik\frac{(x^2+y^2)}{2q(z)}}$ solves this equation. Find *q(z)* using the condition $f(0) = Ce^{-\frac{(x^2+y^2)}{w_0^2}}$, where *C* is a normalization constant.

4) Use Mathematica to plot *w(z)*, where $\left|f(z)\right|^2 \propto e^{-\frac{2(x^2+y^2)}{w(z)^2}}$, *i.e.w(z)* is the width of the beam at position *z*.

5-8) BD Chapter 2 #1-4