Phys 448 HW 7

1) BD 6.1
2) BD 6.2
3) An atom has two states $|g\rangle$ and $|e\rangle$, resonantly (*i.e.* the frequency of the light is equal to the Bohr frequency between the two states) coupled by light, giving an effective interaction $V = \frac{\varepsilon}{2}|g\rangle\langle e| + \frac{\varepsilon}{2}|e\rangle\langle g|$. Find the eigenvectors and eigenvalues of the light-atom system. Find the wavefunction as a function of time, for an arbitrary initial wavefunction, that is find the time evolution matrix *U(t)* such that $\psi(t) = U(t)\psi(0)$. Show that after a time $\varepsilon t = \pi$ (a "π" pulse) an initial state $\psi(0) = |g\rangle$ evolves to $\psi = -i|e\rangle$ and that after a 2π pulse $\psi = -|g\rangle$. How long must you wait for $\psi(t) = \psi(0)$?
4) With the light turned off, the effective atomic Hamiltonian is $H_0 = \hbar(\omega_0 - \omega)|e\rangle\langle e|$. Find the evolution matrix $U_0(t)$. Suppose the atom starts in state g. First a $\pi/2$ pulse is applied. Then the light is turned off for a time *T*. Finally, another $\pi/2$ pulse is applied. After this "Ramsey" sequence, what is the probability of finding the atom in state *e*? Plot it as a function of *T*.
5) Suppose now that the above happens to a set of atoms for differing times *T*. This might arise, for example, because the atoms move through a light-free region of space with differing velocities. Suppose the distribution of times is Gaussian, with mean $\bar{T}$ and standard deviation σ_T. Find the transition probability as a function of time. Plot your result for $(\omega_0 - \omega)\bar{T} = 200\pi$, and $\sigma_T\bar{T} = 4$. How might you use this phenomenon to measure $(\omega_0 - \omega)$ very precisely?