Phys 448 HW 9

1) BD 7.4
2) BD 7.9
3) BD 7.10 (worth double)
4) Particle inside a circle. A particle moves freely inside a circle of radius a. Show that the eigenstates of the Hamiltonian are eigenfunctions of L_z. Use this information to rewrite the 2D Schroedinger equation as a single radial differential equation. Show that the solutions to this equation are Bessel functions, *i.e.* $\psi(\rho, \phi) = J_m(k\rho)e^{im\phi}$.
5) Use the remaining boundary condition to express k and E in terms of x_{mn}, defined by $J_m(x_{mn}) = 0$. Notice that your answer for E is reminiscent of a particle in a box of size a. This is no accident; the radial part of the wavefunction must "fit" into the circular well. To get a quantitative comparison of the two situations, define a "quantum defect" μ_{mn} via
$$E = \frac{\pi^2\hbar^2}{2ma^2}\left(n + \frac{m}{2} - \mu_{mn}\right)^2, n = 1,2 \ldots$$
Calculate the μ_{mn} for $m = 0 \ldots 5$ and $n = 1 \ldots 15$. (The FindRoot function in Mathematica is a convenient tool for finding the x_{mn}.) You should find that the μ_{mn} are between 0 and 1 for each case, and for most of the table are close to ¼. Suppose one replaced the true formula by
$$E' = \frac{\pi^2\hbar^2}{2ma^2}\left(n + \frac{m - 1/2}{2}\right)^2, n = 1,2 \ldots$$
Find the largest percentage error obtained by doing this.

This kind of thing happens a lot in quantum systems; the energy eigenvalues are often accurately described by simple formulae that become exact for large values of the quantum numbers and are still remarkably accurate for small quantum numbers. For a Rb atom, for example, all the energy levels can be represented to about a percent accuracy by three quantum defects, one each for the s, p and d levels. If one allows for a small energy dependence of these quantum defects, the accuracy is much greater.