Ammonia w/ time-dep Electric Field

$$H = \begin{pmatrix} -A & -\eta\cos\omega t \\ -\eta\cos\omega t & A \end{pmatrix}$$

Bohr argument: expect interesting things to happen when $\hbar\omega \approx 2A$

If $\eta = 0$, know $\psi = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} a(0)e^{-iE_S t/\hbar} \\ b(0)e^{-iE_A t/\hbar} \end{pmatrix}$

$$= \begin{pmatrix} a(0)e^{iAt/\hbar} \\ b(0)e^{-iAt/\hbar} \end{pmatrix}$$

so, let's try $\psi = \begin{pmatrix} c(t)e^{i\omega t/2} \\ d(t)e^{-i\omega t/2} \end{pmatrix}$

$$i\hbar\langle\psi_S|\dot\psi\rangle = \langle\psi_S|H|\psi\rangle$$

$$i\hbar\left(\dot c + \frac{i\omega}{2}c\right)e^{i\omega t/2} = -Ac\,e^{i\omega t/2} - \eta\cos\omega t\, d\, e^{-i\omega t/2}$$

$$i\hbar\langle\psi_A|\dot\psi\rangle = \langle\psi_A|H|\psi\rangle$$

$$i\hbar\left(\dot d - \frac{i\omega}{2}d\right)e^{-i\omega t/2} = Ad\,e^{-i\omega t/2} - \eta\cos\omega t\, c\, e^{i\omega t/2}$$

$$i\hbar\dot c = -\left(A - \frac{\hbar\omega}{2}\right)c - \eta\cos\omega t\, d\, e^{-i\omega t}$$

$$i\hbar\dot d = \left(A - \frac{\hbar\omega}{2}\right)d - \eta\cos\omega t\, c\, e^{i\omega t}$$

$$e^{-i\omega t}\cos\omega t = \frac{1}{2} + \frac{1}{2}e^{-2i\omega t}$$

so $\dot{c}$ equation has constant term plus very rapidly changing $e^{-2i\omega t}$ term. We assume that averages to zero very quickly

Similar argument for $\dot{d}$ equation

$$\therefore\ i\hbar\dot{c} \approx -\left(A - \frac{\hbar\omega}{2}\right)c - \eta\frac{d}{2}$$

$$i\hbar\dot{d} \approx \left(A - \frac{\hbar\omega}{2}\right)d - \eta c/2$$

This is the same as an effective Hamiltonian

$$H_{eff} = \begin{pmatrix} -A + \frac{\hbar\omega}{2} & -\eta/2 \\ -\eta/2 & A - \frac{\hbar\omega}{2} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \omega-\omega_0 & -\omega_1 \\ -\omega_1 & \omega_0-\omega \end{pmatrix}$$

where $\hbar\omega_0 = 2A$, $\hbar\omega_1 = \eta$

Use Mathematica to solve:

$$P_s(t) = |\langle\psi_s|\psi(t)\rangle|^2 = \frac{\omega_1^2}{(\omega-\omega_0)^2+\omega_1^2}\sin^2\left[\frac{t}{2}\sqrt{\omega_1^2+(\omega-\omega_0)^2}\right]$$

In[68]:= `conj[a_] := a /. Complex[x_, y_] → Complex[x, -y]`

In[80]:= `$Assumptions = {ℏ > 0, ω0 > 0, ω > 0}`

Out[80]= $\{\hbar > 0, \omega 0 > 0, \omega > 0\}$

Ammonia maser time evolution

In[81]:= $\text{heff} = \frac{\hbar}{2}\begin{pmatrix} \omega - \omega 0 & -\omega 1 \\ -\omega 1 & \omega 0 - \omega \end{pmatrix}$; (*Effective Hamiltonian*)

Initial condition is that the molecule is in the antisymmetric state

In[82]:= `psi0 = {0, 1}`

Out[82]= $\{0, 1\}$

In[83]:= `{eval, kets} = Eigensystem[h] // Simplify`

Out[83]= $$\left\{\left\{-\frac{1}{2}\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}\;\hbar,\ \frac{1}{2}\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}\;\hbar\right\},\right.$$
$$\left.\left\{\left\{\frac{-\omega + \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}{\omega 1}, 1\right\}, \left\{-\frac{\omega - \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}{\omega 1}, 1\right\}\right\}\right\}$$

Mathematica puts the kets in rows, not columns, so the transpose needs to be taken
also, it does not necessarily produce normalized kets. The normalization can be taken care of by defining bras as

In[84]:= $\text{bras} = \text{Inverse}[\text{kets}^{\mathsf{T}}]$

Out[84]= $$\left\{\left\{\frac{\omega 1}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}, \frac{\omega - \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}\right\},\right.$$
$$\left.\left\{-\frac{\omega 1}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}, \frac{-\omega + \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}\right\}\right\}$$

In[85]:= $\text{psi} = \text{Sum}\left[\text{kets}[\![j]\!]\ \text{bras}[\![j]\!].\text{psi0}\ e^{-i\,\text{eval}[\![j]\!]\,t/\hbar}, \{j, 1, 2\}\right]$ // Simplify

Out[85]= $$\left\{\frac{e^{-\frac{1}{2} i t \sqrt{\omega^2-2\omega\omega 0+\omega 0^2+\omega 1^2}}\left(-1 + e^{i t \sqrt{\omega^2-2\omega\omega 0+\omega 0^2+\omega 1^2}}\right)\omega 1}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}},\right.$$
$$\frac{1}{2\sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}}\,e^{-\frac{1}{2} i t \sqrt{\omega^2-2\omega\omega 0+\omega 0^2+\omega 1^2}}$$
$$\left.\left(-\omega + \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2} + e^{i t \sqrt{\omega^2-2\omega\omega 0+\omega 0^2+\omega 1^2}}\left(\omega - \omega 0 + \sqrt{\omega^2 - 2\,\omega\,\omega 0 + \omega 0^2 + \omega 1^2}\right)\right)\right\}$$

amplitude to be in the symmetric state

In[86]:= `{1, 0}.psi // ExpToTrig`

Out[86]=
$$\frac{1}{2\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}}\,\omega1\left(\text{Cos}\left[\frac{1}{2}\,t\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}\right] - \mathbb{i}\,\text{Sin}\left[\frac{1}{2}\,t\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}\right]\right)\left(-1 + \text{Cos}\left[t\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}\right] + \mathbb{i}\,\text{Sin}\left[t\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}\right]\right)$$

calculate probability

In[87]:= `conj[%] % // Simplify`

Out[87]=
$$\frac{\omega1^2\,\text{Sin}\left[\frac{1}{2}\,t\sqrt{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}\right]^2}{\omega^2 - 2\,\omega\,\omega0 + \omega0^2 + \omega1^2}$$

which simplifies to

In[77]:=
$$\frac{\omega1^2\,\text{Sin}\left[\frac{1}{2}\,t\sqrt{(\omega - \omega0)^2 + \omega1^2}\right]^2}{(\omega - \omega0)^2 + \omega1^2}$$

In[97]:=
$$\text{Plot}\left[\frac{\omega1^2\,\text{Sin}\left[\frac{1}{2}\,t\sqrt{(\omega - \omega0)^2 + \omega1^2}\right]^2}{(\omega - \omega0)^2 + \omega1^2}\ /.\ \{\omega0 \to 1,\ t \to 1000,\ \omega1 \to \pi/1000\},\ \{\omega,\ .950,\ 1.050\},\ \text{PlotRange} \to \text{All}\right]$$

Out[97]=