Particle in a Sphere

```
SE[f_] := (-ℏ^2/(2 m)) D[f, {r, 2}] + (ℏ^2 l (l + 1))/(2 m r^2) f == energy f
```

$$SE[f\_] := \frac{-\hbar^2}{2\,m} D[f, \{r, 2\}] + \frac{\hbar^2\, l\,(l+1)}{2\,m\,r^2} f == \text{energy } f$$

```
$Assumptions = {k > 0}
```

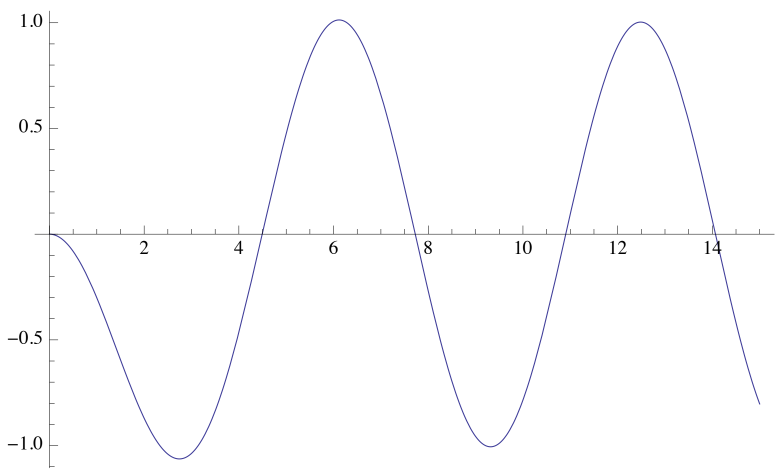
{k > 0}

l=1

```
SE[k r SphericalBesselJ[1, k r] // FunctionExpand] /. l → 1 // Simplify
```

$$\frac{\left(2\,\text{energy}\,m - k^2\,\hbar^2\right)\,(k\,r\,\text{Cos}[k\,r] - \text{Sin}[k\,r])}{m\,r} == 0$$

```
Plot[Cos[x] - Sin[x] / (x), {x, 0, 15}]
```



Find zeros to match boundary condition at r=a

```
Table[FindRoot[x SphericalBesselJ[1, π x] == 0, {x, (i + 1 / 2)}], {i, 5}]
```

{{x → 1.4303}, {x → 2.45902}, {x → 3.47089}, {x → 4.47741}, {x → 5.48154}}

$$\therefore \text{ka} \simeq \pi\,(n_r + 3/2), E = \frac{\pi^2\,\hbar^2}{2\,m\,a^2}\,(n_r + 3/2)^2$$

l=2

```
SE[k r SphericalBesselJ[2, k r] // FunctionExpand] /. l → 2 // Simplify
```

$$\frac{\left(2\,\text{energy}\,m - k^2\,\hbar^2\right)\,\left(3\,k\,r\,\text{Cos}[k\,r] + \left(-3 + k^2\,r^2\right)\,\text{Sin}[k\,r]\right)}{m\,r} == 0$$

```
Table[FindRoot[x SphericalBesselJ[2, π x] == 0, {x, (i + 1)}], {i, 5}]
```

{{x → 1.83457}, {x → 2.89503}, {x → 3.92251}, {x → 4.93845}, {x → 5.94891}}

$$\therefore \text{ka} \simeq \pi\,(n_r + 1 + l/2),\ E = \frac{\pi^2\,\hbar^2}{2\,m\,a^2}\,(n_r + 1 + l/2)^2$$

```
Plot[x {SphericalBesselJ[0, x], SphericalBesselJ[1, x], SphericalBesselJ[2, x]},
 {x, 0, 15}]
```